Tractive Effort of Steam Locomotives
(Locomotive Ratios—II)

by A. I. LIPEZ, SCHENECTADY, N. Y.

In a paper, presented before the A.S.M.E., December, 1932, the author developed constants for a new method of figuring horsepower and tractive effort of steam locomotives based on boiler evaporation and number of revolutions of the driving wheels for use in connection with modern locomotives. This method was verified on a number of locomotives, for which it gave accurate results in accordance with test data.

In the present paper the author discusses the influence of the size of cylinders and shows that for locomotives with certain proportions between boiler and cylinder dimensions modifications of previously recommended factors are necessary. These modifications are verified for a number of locomotives with comparatively large cylinders. In conclusion he develops a convenient formula based on both boiler evaporation and cylinder dimensions, applicable to all modern locomotives, whether of the long- or limited cut-off type (with small or large cylinders).

Introduction and Recapitulation

In his paper presented at the Annual Meeting, New York, N. Y., Dec. 5-9, 1932, of the American Society of Mechanical Engineers, the author of the present paper suggested a method of evaluating horsepowers and tractive efforts of steam locomotives by means of certain moduli, which he designated

As the method of 1932. It was recommended that for indicated tractive effort \( T_r \), the following formula be used:

\[
T_r = M_t \frac{E_c}{D} \quad \text{[1]}
\]

in which

- \( E_c \) = the Cole boiler evaporation determined by the American Locomotive Company’s Handbook, Edition of 1917, p. 59, with additions as given in Tables 8 and 9 of the 1932 paper
- \( D \) = the diameter of driving wheels, in inches
- \( M_t \) = a modulus, the values of which can be taken from a table given in the paper and reproduced here (Table 1).

<table>
<thead>
<tr>
<th>Revolutions per minute (n)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locomotives with feedwater heaters:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_p \times 1000 )</td>
<td>26.0</td>
<td>43.1</td>
<td>52.0</td>
<td>54.0</td>
<td>51.0</td>
</tr>
<tr>
<td>( M_t )</td>
<td>65.6</td>
<td>54.4</td>
<td>43.7</td>
<td>34.1</td>
<td>25.7</td>
</tr>
<tr>
<td>Locomotives without feedwater heaters:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_p \times 1000 )</td>
<td>24.3</td>
<td>40.3</td>
<td>48.0</td>
<td>50.5</td>
<td>47.7</td>
</tr>
<tr>
<td>( M_t )</td>
<td>61.8</td>
<td>50.8</td>
<td>40.8</td>
<td>31.8</td>
<td>24.0</td>
</tr>
</tbody>
</table>

For horsepower \( P_t \), a corresponding formula was proposed, namely

\[
P_t = M_p \frac{E_c}{S} \quad \text{[2]}
\]

with modulus \( M_p \), also given in a table of the paper. (See Table 1.) The latter formula was necessarily dependent upon formula [1] in accordance with the known relation

\[
P_t = T_r S/375 \quad \text{[3]}
\]

which meant that moduli \( M_t \) and \( M_p \) were interconnected by a formula

\[
M_p = M_t \frac{n}{126,050} \quad \text{[4]}
\]

in which

- \( n \) = the number of revolutions per minute
- \( S \) = the train speed of the locomotive in miles per hour.

In the present paper only tractive efforts will be considered for the reason that horsepowers can always be figured on the basis of formula [3], if tractive efforts are known, or on the basis of formulas [2] and [4].

The discussion which followed the presentation and publication of the paper was, on the one hand, gratifying in that it showed the interest of railroad engineers in the subject, and, on the other hand, was interesting from the point of view of the practicability of the recommended method. A number of discussers agreed that the new method was practical and was giving reliable results, although some thought that it was losing the simplicity of the Cole method.

Especially interesting was the contribution made by H. S. Vincent, which was to the effect that an algebraic formula, representing a modification of the formula previously suggested by W. F. Kiesel, would give more accurate results in application to certain types of locomotives which the author did not consider

\[ \text{For speeds n not given in Table 1, moduli } M_t \text{ and } M_p \text{ can be figured on the basis of formulas [14] and [15], and Figs. 3 and 5, of the 1932 paper.} \]
in his paper for the reason that he had no information on these locomotives.  

In analyzing this question the author found that the fact that the 1932 method had given such good coincidence in a great number of cases, while in some enumerated by Mr. Vincent, it did not, was due to two reasons: First, to the fact that in the latter cases, when comparisons between locomotives were made, the rates of the utilization of the boiler, commonly known as "boiler forcing," and the locomotive efficiencies were different from what they were supposed to be according to the 1932 method; and second, to the difference in the ratios between boiler and cylinder dimensions. It turned out that the modern locomotives for which data were available and which the author took as a basis for his theory all had a rather high ratio of heating surface in square feet to the total volume of all simple-expansion cylinders in cubic feet, varying from 240 to 280, while the locomotives analyzed by Mr. Vincent had comparatively large cylinders (some of the locomotives being of the so-called "limited cut-off type") and low ratios of heating surface to cylinder volume, which fluctuated between 170 and 220. Therefore, the author found it necessary to consider a new variable—that of the cylinder volume—if all types of locomotives are to be covered by one method for figuring tractive effort.

The influence of these two causes will be discussed in the present paper, the necessary conclusions will be drawn, and new recommendations, designated as those of 1934, made. In the 1932 paper the soundness of the method was verified by applying it to six modern locomotives for which test data were available. The locomotives were the following:

- New York Central, 4-6-4 (Class J-1a)
- New York Central, 4-8-2 (Class L-2)
- Lehigh Valley, 4-8-4 (Class 5100)
- Lehigh Valley, 4-8-4 (Class 5200)
- Timken locomotive, 4-8-4 (No. 1111)
- Boston & Albany, 2-8-4 (Class A-1)

The curves according to the method, in comparison with the test curves, called performance curves, were shown on pages 14, 15, and 34 of the paper and printed discussions. The curves were plotted on the basis of data obtained from road tests, which, in the opinion of the author, better represent every-day locomotive performance. It will be shown in this paper that the new (1934) recommendations do not alter materially the 1932 curves and do not disturb the agreement between them and the performance-test curves. On the other hand, all the large-size-cylinder locomotives, reviewed by Mr. Vincent in his discussion of the author's 1932 paper, as well as in his subsequent article in the Railway Mechanical Engineer, will be examined in the present paper together with other locomotives on which data had become available lately. These locomotives are:

- Missouri Pacific, 2-8-2 (3-cylinder)
- Texas & Pacific, 2-10-4 (Class I-1)
- Texas & Pacific, 2-10-2 (Class G-1b)
- Atchison, Topeka & Santa Fe, 2-10-4 (No. 5000)
- Pennsylvania R.R., 2-10-0 (Class L-1a)
- German State Railways, 4-6-2 (No. 10201)
- German State Railways, 2-8-0 (No. 562131)
- German State Railways, 2-10-0 (No. 43001)

It was also shown in the 1932 paper that it is practically impossible to offer a theoretical formula for the tractive effort of a locomotive based on scientific premises. Even if a theory could embrace satisfactorily all relations between various phenomena that take place in a locomotive, the formula would have to be adjusted afterward on the basis of test data; in other words, the formula must necessarily be empirical. The author, in his previous paper, pointed out the reasons for an empirical method based on boiler evaporation, rather than cylinder dimensions. The modifications which are offered in this paper still have the underlying basis of boiler evaporation as the source of locomotive power. However, it will be shown that, if desired, an empirical formula for the locomotive tractive effort can be devised, which, although not quite accurate, may be useful for practical purposes and satisfy the tastes of engineers who prefer formulas to charts.

**Road and Stationary Locomotive Tests**

It was brought out during the discussions of the 1932 paper and in the author’s closure that the performance tractive effort according to the suggested method should not be considered as the maximum possible tractive effort. When the locomotive is in perfect condition and is tested under constant conditions of work—cut-off, speed, load—the test will show higher figures than when the regular road locomotive is tested with a commercial train on an undulated profile and at variable conditions of work. This is especially true for this country, where locomotives are tested with regular revenue trains, although it is not true that under no conditions can road tests yield accurate data. In other countries methods have been worked out with the object of holding the load and speed constant on an undulated profile. In this country no special methods are in use, except only that inaccuracies introduced by acceleration and grade resistance are taken into account and proper corrections are made. Furthermore, in some cases, a more or less uniform profile is chosen, at least for a considerable portion of the test run, and observations at intervals as short as possible are usually made.

Especially, the American methods of testing locomotives cannot be used for figuring water consumption per unit of work, or locomotive efficiencies, because the figures which are obtained for water or coal consumption are known only for the whole run and necessarily must be of an average character. However, in so far as the average tractive efforts and speeds during the short intervals referred to are concerned, the American road methods are fairly accurate, if the necessary corrections for grade and acceleration are made, although necessarily they are lower than the stationary-plant test data. The average tractive efforts and speeds obtained during these short intervals, ordinarily of five-minute duration, form the basis of the performance tractive-effort curve.

In stationary tests, on the other hand, higher figures are being obtained for the reason that the tests are made under constant load and speed conditions and under the supervision of skilled engineers specially trained for testing locomotives at a stationary plant. The limit of power can, under these conditions, be easily reached, which is not always possible to get from the average crew under every-day operating conditions in regular road service. Therefore, it is natural that stationary tests will show higher figures than road performance tests. They will approach the so-called capacity test figures, which are usually about 20 to 25 percent higher than performance figures, as can be seen from Figs. 10 and 11, pages 14 and 34, of the 1932 paper. Consequently, when locomotives are compared, either one method of testing (stationary plant), or the other method (road tests), should be used. When data from both stationary and road tests are compared and conclusions are drawn, only confusion can result.

A very good illustration of the difference between the tractive-effort curve according to the 1932 method and the corresponding
A test curve is offered by the Pennsylvania Railroad 4-8-4 locomotive, Class M. The first engine of this class built (No. 4700), had two 27 by 30-in. cylinders, 72-in. driving wheels, 250-lb working pressure, a boiler with 114 \( \frac{2}{3} \) by \( \frac{1}{2} \)-in. tubes and 200 3\( \frac{1}{4} \)-in. flues, type E superheater, and a feedwater heater. The evaporative heating surface of the locomotive (water side) was 4904 sq ft. The locomotive was thoroughly tested at the Altoona testing plant in 1924 and the tests were not all that the railroad desired. The boiler was redesigned and the number of tubes was changed; 120 2\( \frac{1}{4} \)-in. tubes and 170 3\( \frac{1}{4} \)-in. flues were applied, the steam space was raised, and the total evaporative heating surface (water side) was reduced to 4696 sq ft. The machinery remained substantially the same. It became known as the M-1 class. A locomotive of this class, No. 6872, was tested at the Altoona stationary plant in 1929. The highest tractive effort figures obtained at these tests are shown in Fig. 1 by dots, with corresponding test numbers. The test dots are connected in one continuous line marked A. The 1932 curve is also drawn in Fig. 1.

The discrepancy between these two curves is very pronounced: At low speeds it amounts to 21 per cent; at high speeds to 36 per cent. However, it would not be fair to compare these two curves and to blame the 1932 method, because the two curves are of different character. The 1932 curve is the performance curve which would be safe to expect in ordinary road service of this locomotive, while the Altoona curve is the maximum test curve which can be obtained when the locomotive is working at its limit of capacity, under the best possible test conditions and under the supervision of specially trained skilled experimenters. The main difference between these two curves will be clearer if the efficiencies are considered.

The 1932 curves were based on the performance data of the New York Central J-1 and L-2 locomotives and on similar data of other locomotives, already enumerated. Under the more or less constant conditions of work the New York Central test reports showed fuel consumptions which correspond to an overall thermal efficiency of about 6 per cent. This represents a normal locomotive utilization when the power is not pushed to the limit. The high points obtained during stationary tests indicate different conditions. The overall thermal efficiencies of the M-1 locomotive from the Altoona tests are shown in Fig. 1 for every test separately. They are also repeated in Table 2, which shows that the average overall efficiency of the locomotive, when worked at the highest tractive-effort curve, is only 4.16 per cent. This figure is underlined in Fig. 1. The corresponding boiler efficiencies, when the locomotive is worked very hard and the boiler is forced, are also given in Table 2.

\[
\text{TABLE 2 EFFICIENCIES, LOCOMOTIVE M-1, NO. 6872}
\]

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Overall efficiency, per cent</th>
<th>Boiler efficiency, per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>161-A</td>
<td>3.8</td>
<td>52.5</td>
</tr>
<tr>
<td>181-A</td>
<td>4.2</td>
<td>51.3</td>
</tr>
<tr>
<td>157-A</td>
<td>3.7</td>
<td>46.6</td>
</tr>
<tr>
<td>167-A</td>
<td>3.7</td>
<td>45.1</td>
</tr>
<tr>
<td>151-A</td>
<td>4.3</td>
<td>48.1</td>
</tr>
<tr>
<td>178-A</td>
<td>4.8</td>
<td>60.7</td>
</tr>
<tr>
<td>173-A</td>
<td>4.6</td>
<td>51.5</td>
</tr>
<tr>
<td>Average</td>
<td>4.16</td>
<td>49.0</td>
</tr>
</tbody>
</table>

In order to verify whether the 1932 curve corresponds to a higher efficiency, all the remaining tests, outside of the high-point tests of the locomotive, were analyzed. Tests with tractive efforts of values close to those represented by the 1932 curve gave efficiencies from 4.7 to 6.1 per cent. In Fig. 1 the averages especially interesting in view of the importance which was attached to it by Mr. Vincent in his discussion. All data have

\[
\text{TABLE 3 LOCOMOTIVE L-1a, NO. 4358}
\]

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Overall efficiency, per cent</th>
<th>Boiler efficiency, per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>5940</td>
<td>4.7</td>
<td>56</td>
</tr>
<tr>
<td>5939</td>
<td>5.1</td>
<td>55</td>
</tr>
<tr>
<td>5933</td>
<td>5.6</td>
<td>54</td>
</tr>
<tr>
<td>5929</td>
<td>4.0</td>
<td>42</td>
</tr>
<tr>
<td>5935</td>
<td>3.6</td>
<td>41</td>
</tr>
<tr>
<td>5935</td>
<td>3.8</td>
<td>39</td>
</tr>
<tr>
<td>6123</td>
<td>4.1</td>
<td>38</td>
</tr>
<tr>
<td>5923</td>
<td>4.5</td>
<td>45</td>
</tr>
<tr>
<td>5972</td>
<td>4.4</td>
<td>49</td>
</tr>
<tr>
<td>5972</td>
<td>5.1</td>
<td>35</td>
</tr>
<tr>
<td>Average</td>
<td>4.29</td>
<td>45.2</td>
</tr>
</tbody>
</table>

been taken from P.R.R. Bulletin No. 32, which gives complete information on the Altoona tests of this locomotive. Similar to
average tractive efforts of all tests, except the highest, at a certain speed, recorded at the Altoona tests. The underlined figures are the average efficiencies of all the tests represented by the corresponding tractive-effort point. It can be seen that these efficiencies vary from 6.20 to 7.20 per cent, as compared with 4.29 per cent for the maximum tractive-effort curve.

In order to make the charts complete, the Vincent curves are also plotted.

In addition to the two main reasons, namely, the extreme forcing of the boiler to its limit, and the advantages of stationary tests, the differences between road and stationary plant tractive efforts are also sometimes due to the difference in locomotive design in general, and in the drafting arrangements in particular, but the two reasons cited are the most important and can explain the disagreements which are found between tractive-effort curves when a comparison of curves of different nature is made.

This does not mean, however, that high tractive efforts with forcing of the boiler and low efficiencies cannot be obtained during road tests and even regular road performance. If the necessity arises, the engineer will be able to force his boiler to the limit and get high tractive efforts. In other words, in locomotives with large cylinders, the Pennsylvania L-1s locomotive is the best example of such a case, because the ratio of boiler heating surface to cylinder volume is 176.4, compared with 240-280, as in many modern locomotives (in the Lehigh Valley locomotive No. 5100, referred to above as having a 100-per cent boiler, this ratio is 272.7). These figures are given here only as an illustration, because the proper relation between boiler capacity and cylinder volume will be discussed later in a more detailed way.

When a locomotive of this large-size-cylinder type starts out with a train at very low speeds, with the maximum cylinder tractive effort, which ordinarily is very close to the rated and to the adhesion tractive effort, as it has been pointed out by the author in his 1932 paper, the ratio of boiler heating surface to cylinder volume is 176.4, compared with 240-280, as in many modern locomotives (in the Lehigh Valley locomotive No. 5100, referred to above as having a 100-per cent boiler, this ratio is 272.7). These figures are given here only as an illustration, because the proper relation between boiler capacity and cylinder volume will be discussed later in a more detailed way.
**Boiler Adequacy**

One of the most important proportions in a locomotive is the ratio between boiler dimensions and cylinder volume. This was recognized long ago, and in 1897 the American Railroad Master Mechanics' Association recommended that for bituminous coal the ratio of the evaporative heating surface in square feet to the volume of two cylinders in cubic feet be 200. With the advent of superheated steam, the size of cylinders has necessarily increased. At the same time, boilers became more powerful in relation to cylinders, so that at present the ratio in modern locomotives, as has been stated, sometimes reaches 280.

On the other hand, about fifteen years ago the idea of large cylinders and shorter cut-offs attracted some railroad engineers in this country to the extent that, as we saw in the I-1s locomotive, this ratio went back to 176.4.

Mr. Cole introduced the term "boiler percentage," which represents a ratio of boiler capacity measured in horsepower to cylinder horsepower, the first being equal to boiler evaporation per hour divided by steam consumption of 28.0 lb per hp-hr for saturated steam, and 20.8 lb for superheated steam, and the second representing the cylinder horsepower according to his method figured at a piston speed of 1000 ft per min, at which speed the maximum Cole horsepower is obtained.

It is very difficult to defend this conception of boiler percentage for the reason that neither one nor the other horsepower represents correct figures. The steam consumption of 20.8 lb per hp-hr is too high for modern locomotives with superheated steam and feedwater heaters; but more important than this is the fact that the cylinder horsepower, apart from the boiler evaporation, is a very indefinite term. If, at the speed of 1000 ft per min, corresponding approximately to 200 rpm, or, with 70-in. driving wheels, to a track speed of 41.7 mph, the horsepower reaches a certain figure, corresponding to a cut-off of, say, 25 per cent, there is no reason why the cut-off cannot be increased within certain limits, provided the boiler evaporation permits; in other words, the term "cylinder horsepower," apart from the boiler horsepower, is a misconception. The former depends upon the latter and, therefore, an independent ratio between the two cannot be obtained.

In the author's opinion, it would be more logical to compare cylinder and boiler tractive efforts at a speed at which both terms have definite meanings, and his recommendation would be to make that comparison at a speed close to the point where the maximum cylinder, or rated, tractive effort intersects the boiler tractive effort, as defined in his first paper.

The point of intersection of these two curves will depend upon boiler capacity, as it was shown in Fig. 1 of the author's 1932 paper, reproduced here as Fig. 4. At a certain point A on the maximum cylinder-tractive-effort curve, corresponding to, say, 85 per cent cut-off, the latter will have to be reduced from the maximum to, say, 81 per cent, on account of the limitation of boiler capacity. The tractive effort will follow the line AB, and point A will be the above referred to intersection point. Point A' is a similar point in case the boiler evaporation is greater. The tractive effort will then follow line A'B'. In the first case, speed S1 will be attained at the maximum cylinder tractive effort, while in the second case a greater speed, corresponding to point A', can be reached.

The upper part of the maximum cylinder tractive effort is very close to the rated tractive effort, and instead of the maximum cylinder tractive effort, the rated tractive effort can be considered. It is evident that, depending upon the size of the boiler, the maximum speed at the rated tractive effort will vary.

Thus, a much better conception of the ratio between

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*Railway Mechanical Engineer, November, 1933, p. 390.*

and boiler power will be obtained, if we should agree that the intersection of the rated and boiler tractive efforts should be at a certain speed, either constant or varying, depending upon the service of the locomotive. At present, in view of the fact that many road freight locomotives are being designed for high speeds with drivers approaching the sizes of driving wheels of passenger locomotives, the author suggests making this speed equal to 50 rpm, which, with drivers of 69 in., represents a track speed of 10.4 mph, while with drivers of 80-in., this corresponds to a track speed of 12.1 mph. In view of this, the author suggests to measure the proper relation between boiler and cylinder di-

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![Fig. 4](image_url)
Thus

\[ a_s = \frac{M_{in}E_1}{1100.7 \alpha V_p} \]  

Introducing a new locomotive constant \( K = \frac{E_1}{V_p} \), we can say that

\[ a_s = \frac{M_{in}K}{1100.7 \alpha} \]  

For modern locomotives with feedwater heaters, \( M_{in} = 65.6 \), while for locomotives without feedwater heaters, \( M_{in} = 61.3 \). Thus, for locomotives with feedwater heaters

\[ a_s = 0.0596K/\alpha \]  

and for locomotives without feedwater heaters

\[ a_s = 0.0557K/\alpha \]

If a boiler adequacy of 1.0 is desired, \( K \), for locomotives with feedwater heaters, must be

\[ K_1 = \alpha/0.0596 \]

and for locomotives without feedwater heaters

\[ K_2 = \alpha/0.0557 \]

Thus, \( K \) turns out to be a very important locomotive constant, on which the correct proportion between boiler and cylinder dimensions depends. We shall call it hereafter "locomotive characteristic." It is determined by

\[ K = \frac{E_1}{V_p} \]

In a well-proportioned locomotive with \( a_s = 1 \), \( K \) depends only upon the maximum mean indicated pressure ratio \( \alpha \). For long cut-off locomotives with \( a_{max} = 0.85 \), \( K \) will thus be 14.26. For locomotives with shorter cut-offs, \( K \) will be slightly smaller in proportion to \( \alpha \).

It should not be thought, however, that \( K \) must be an absolutely rigid figure. The variation in this figure means, as it was shown above, a variation in the highest speed at which the locomotive can run with its maximum cylinder tractive effort (close to its adhesion limit). For passenger locomotives, this ratio should be higher (more boiler capacity), while for freight locomotives this ratio may be slightly lower. For what are called at present high-speed freight engines, this ratio should be about 14.26. It is interesting to note that the very successful modern long cut-off locomotives have the following characteristics \( K \):

- New York Central, 4-6-4 (Class J-1a).....................15.28
- New York Central, 4-8-2 (Class L-2)...................13.25
- Lehigh Valley, 4-8-4 (Class 5100)......................14.2
- Lehigh Valley, 4-8-4 (Class 5200)......................14.3

For the Boston & Albany 2-8-4 (class A-1) locomotive with a maximum cut-off of 60 per cent, for which \( a_{max} = 0.78 \), \( K = 12.27 \), which is not so far off from the ideal \( K \), in accordance with formula \( [9a] \), namely \( K = 0.78/0.0596 = 13.09 \). In other words, this locomotive, although of the limited cut-off type, with large cylinders, has not been greatly different from the conventional locomotive, and the cylinder sizes were very well chosen. This explains why in the author's first paper the tractive effort of locomotive A-1 was shown to be in accordance with the 1932 method, just like any long cut-off locomotive.

As to other limited cut-off locomotives, the cylinders were not made in proportion to the boiler; they were further enlarged beyond the size required for the limited cut-off feature, although sometimes it is being stated that "when the maximum cut-off is shortened, the cylinder is correspondingly increased in diameter." Very often it is being increased much more than correspondingly and than necessary. So, for instance, the Texas & Pacific I-1 locomotive, with a maximum cut-off of 60 per cent, has a \( K \) of only 10.97. The Atchison, Topeka & Santa Fe No. 5000, with similar 60 per cent maximum cut-off, has a \( K \) of only 9.92, and the Pennsylvania I-1s locomotive, with a maximum cut-off of 55 per cent, has a still smaller \( K \), namely, 8.53. These \( K \)s have been made much smaller than what the relatively small change in \( a_{max} \) would require, as it is reflected in the values of boiler adequacy.

In Table 4 some principal dimensions and locomotive ratios are given for all locomotives which have been discussed in the author's previous paper, in Mr. Vincent's article in the *Railway Mechanical Engineer*, November and December, 1933, and in the present paper. The locomotives are listed in descending order of \( K \) (in last column). From this table it can be seen that:

\[ 1 \] *Railway Mechanical Engineer*, December, 1933, p. 432.

<table>
<thead>
<tr>
<th>No.</th>
<th>Locomotive</th>
<th>Cyl. dim.</th>
<th>Heating surface, in.</th>
<th>Boiler pressure, lb per sq ft</th>
<th>Cola evaporation, lb per hr</th>
<th>Driving wheel diam, in.</th>
<th>Boiler adequacy, %</th>
<th>Boiler constants</th>
<th>Cyl. vol., cu ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Penn. E-6a, No. 89</td>
<td>22</td>
<td>2685</td>
<td>205</td>
<td>39,870</td>
<td>A</td>
<td>No</td>
<td>0.85</td>
<td>61.3</td>
</tr>
<tr>
<td>2</td>
<td>N. Y. C. J-1</td>
<td>25</td>
<td>2685</td>
<td>205</td>
<td>39,870</td>
<td>B</td>
<td>Yes</td>
<td>0.85</td>
<td>56.6</td>
</tr>
<tr>
<td>3</td>
<td>Penn. R-2a No. 877</td>
<td>24</td>
<td>2685</td>
<td>205</td>
<td>39,870</td>
<td>E</td>
<td>Yes</td>
<td>0.85</td>
<td>56.6</td>
</tr>
<tr>
<td>4</td>
<td>Penn. E-6a No. 51</td>
<td>23/1</td>
<td>2977</td>
<td>205</td>
<td>40,630</td>
<td>D</td>
<td>No</td>
<td>0.85</td>
<td>56.6</td>
</tr>
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<td>5</td>
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<td>A</td>
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<td>27</td>
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<td>61,511</td>
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<td>240</td>
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<td>2756</td>
<td>227</td>
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<td>0.83</td>
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<td>57,689</td>
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ning with the Pennsylvania E-6s locomotive, with a $K$ of 16.88, down to the Pennsylvania I-1s locomotive, with $K = 8.53$, the range of variation in boiler adequacy is very wide—ratio between extreme values 1.63—more than could be expected, even if the difference in the kind of service of the locomotives is taken into consideration.

In addition to column 11 of Table 4, in which the boiler adequacy for each boiler is given, for comparative purposes Cole boiler percentages are also shown in column 12. Credit must be given to Mr. Cole for his conception of boiler percentage, which, although theoretically not quite correct, differs very little from the more accurate boiler adequacy, and has a close range variation with the latter—1.88 as compared with 1.63, as shown.

It is interesting to note that a constant very similar to $K$, in which the boiler evaporation in the numerator is replaced by the evaporative heating surface in square feet, namely, $\delta = \frac{H_e}{V_p S}$ may be also very useful for comparative purposes. It is given in column 14 of Table 4, and it can be seen that for the first thirteen locomotives, including the Pennsylvania M-1 and the Boston & Albany A-1, $\delta = 0.995$, or more, while for all others, including limited cut-off locomotives (except Nos. 14 and 15, which are of the three-cylinder type and necessarily have a large cylinder volume), $\delta$ is less than 0.86. There is a gap in $\delta$ of these two groups, one having approximately $\delta = 1.0$ and more, while the other has $\delta = 0.86$ and less. When the boiler evaporation is not quite known, it might be advantageous for the first approximation to figure the cylinder volume $V$ on the basis of evaporative heating surface, assuming $\delta = 1.0$, which is equivalent to

$$V = \frac{H_e}{p_o} \quad \text{[11]}$$

It should be later checked on the basis of $K$.

**Tractive Effort of Large-Size-Cylinder Locomotives**

It has been shown before that the 1932 method gives accurate results for certain locomotives of modern type. Here it has been pointed out that these locomotives are all of the large-boiler type, with locomotive characteristic $K$ of about 12-16. It was also found that when a boiler adequacy of 1.0 is required, $K$ should be 14.26 for locomotives with feedwater heaters.

It was also brought out in the foregoing discussion that when a locomotive has comparatively large cylinders and a small characteristic $K$, the boiler at low speeds is usually overstressed. This can be done because, as it will be remembered from the 1932 paper, the boiler evaporation was assumed to be, at low speeds, much below $E_r$—the Cole evaporation figure; at 50 rpm it was only 65 per cent of the Cole figure. Many investigators claim that the boiler evaporation is more or less constant and that its maximum, if there are any fluctuations, can be obtained almost at any speed. This is true, if forcing of the boiler and low boiler efficiencies are permitted; in other words, the fact that it is possible to force the boiler at low speeds above the 1932 values is evident on the basis of premises used for the 1932 method.

There is not enough available information from tests to indicate the relation between the limit of forcing at low speeds and locomotive characteristic $K$. The author has, therefore, followed the same method which he pursued in his 1932 paper, namely, he investigated the test results of existing locomotives and tried to find whether there were any simple and consistent relations between the test data and the locomotive principal dimensions. He did find some, and he verified his findings on all locomotives for which information was available. Good coincidence was found; this permits the claim, with a sufficient degree of certainty, that the findings are correct.
ing in mind that these test figures have been obtained at the Altoona testing plant, the discrepancy can be easily explained in the light of previous discussion.

It has been argued by Mr. Vincent that the cylinder tractive effort at low speeds should be taken as an inclined line. In the author’s closure to his 1932 paper he admitted that this is in principle correct, if the highest mean-indicated-pressure ratios at low speeds, which ratios may go up to 0.93, are used; when the rated tractive-effort value of 0.85 and less is used, the horizontal line is more conservative, and sufficiently accurate. The rule which the author suggests now is to connect the tractive-effort point at 50 rpm, either modified, or not, depending upon value of K, with a point corresponding to the rated tractive-effort value at zero speed, if the latter value is higher than the 1934 value for 50 rpm. The following example will show a case when it is lower. In the Missouri Pacific locomotive, the former is 64,890 lb, and we receive the inclined line TV, as desired by Mr. Vincent.

Fig. 7 represents Texas & Pacific 2-10-4 I-1 engine, which has a 60 per cent limited cut-off. As K is equal to only 10.07, similar modifications have to be made, in accordance with Fig. 5. The 1934 curve is thus obtained. The $T_{100}$ value in this case is 69,852, and the 1934 $T_r'$ value is 84,700 lb, with a mean indicated pressure ratio of 0.78. In this case the author suggests that a straight line should be drawn equal to the rated tractive effort $T_r$ until it intersects the 1934 curve, although in principle he would not object to an inclined line, especially for a limited cut-off locomotive. The 1934 curve $T_r'$, combined with $T_r$, will determine the tractive-effort line of the locomotive.

The crosses shown on the chart are test values, as given in Mr. Vincent’s article in the *Railway Mechanical Engineer*. It can be seen that the coincidence of both $T_r$ and $T_r'$ curves with test data is very satisfactory and that there is no necessity for an inclined cylinder tractive effort and a transition line, as suggested by Mr. Vincent.

Fig. 8 represents another Texas & Pacific locomotive, class G-1b, with a still smaller K (10.63), although it is a full cut-off engine. Although this is not a limited cut-off locomotive, but due to the fact that K is low, only 10.63, even lower than in many limited cut-off engines, the modification must be made, and the 1932 curve has to be raised, with the assistance of Fig. 5, to that shown as 1934 on Fig. 8. $T_{150}^\prime$ (61,900 lb) is less than $T_r$ (67,698) and, therefore, an inclined curve for cylinder tractive effort, for speeds between zero and 50 rpm, is drawn.

The crosses again represent test data given by Mr. Vincent in his *Railway Mechanical Engineer* article. It can be seen that these crosses agree very nicely with the inclined line for low speeds, as well as with the $T_r'$ curve.

It is interesting to note that both Texas & Pacific locomotives

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*December, 1933, p. 430, Fig. 11.

*November, 1933, p. 394, Fig. 8.*
are oil-burners, and that no substantially increased powers had been obtained with oil compared with coal.

Fig. 9 pertains to the Atchison, Topeka & Santa Fe limited cut-off, 2-10-4, locomotive No. 5000, for which \( K \) is 9.92. The modified tractive effort is marked 1934, and the crosses, representing test values, are taken from Mr. Vincent's article in the Railway Mechanical Engineer.\(^{11}\) They lie higher than the 1934 curve, which, in the opinion of the author, is probably due to the fact that the locomotive had been stressed to the capacity limit. If all data, and especially the efficiencies of the locomotive for the high points, were given, this could be proved definitely.

Figs. 10, 11, and 12 represent German two-cylinder locomotives of types and numbers as given in the figures. These engines are German standard locomotives, thoroughly tested at a constant evaporation of 57 kg of steam per hour and per square meter of inside heating surface, corresponding to 10.7 lb of steam per hour per square foot of outside heating surface, this compared with evaporations of 12 to 13 lb on the basis of Cole figures, which were assumed for the 1932 and 1934 methods. The German tests were described by Professor Nordmann in the Organ für die Fortschritte des Eisenbahnuensens, and the test curves are plotted on the basis of his curves.\(^{12}\)

As the figures of Professor Nordmann's article give the horsepower and not the tractive efforts, the horsepower curves were shown in Figs. 10 to 12 of the present paper and the tractive efforts were calculated from the horsepowers. The 1934 tractive-effort and horsepower curves are also shown, and it can be seen that the agreement is very good.

The efficiencies of the German locomotives at different speeds varied between 7 and 9 per cent,\(^{18}\) and this further confirms the author's statement that the 1934 curves correspond to reasonable locomotive efficiencies—the Cole evaporations are higher than the German constant figure of 10.7 lb per hr per sq ft of heating surface, but not as high as in some Altoona tests, and, therefore, the efficiencies which correspond to the 1934 curves are somewhere between 9 and 4 per cent, probably 6 to 7.

As to the last locomotive in Table 4, with the smallest \( K \), the Pennsylvania 2-10-0 I-1-8 engine, the curves had been already shown in Fig. 2 and discussed.\(^{14}\)

It has been stated before that locomotives with \( K \) more than 14.26 do not require any modifications. This has been shown on Fig. 3 for Lehigh Valley locomotive No. 5100, with a \( K \) of 14.2 and can be verified on all long cut-off locomotives discussed in the author's 1932 paper. For instance, in Fig. 13 the 1932 and performance curves for the New York Central 4-6-4 J-1 locomotive are reproduced from Figs. 11 and 11-A of the author's 1932 paper. Likewise, Fig. 15 gives the 1932 and performance curves for Lehigh Valley locomotive No. 5200 reproduced from Figs. 14 and 14-A of the previous paper. For both locomotives \( K \), as can be seen from Table 4, is higher than 14.26, and the 1932 and 1934 curves coincide.

Fig. 14 shows corresponding curves for the New York Central 4-8-2 L-2 locomotive. This locomotive has a \( K \) of 13.25, and the modification required in accordance with Fig. 5 is very slight, as can be seen from Fig. 14. The performance curve on the latter figure does not differ much from the 1934 line, which has been heightened as compared with the 1932 line for speeds between 50 and 200 rpm.

The modifications and Fig. 5 referred to were derived by studying the performance of locomotives with feedwater heaters. Nevertheless, they apply also to locomotives without feedwater heaters, because the increase in tractive effort at low speeds due to the sizes of cylinders does not depend upon whether the

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\(^{11}\) November, 1933, p. 392, Fig. 4.

\(^{12}\) May 15, 1930, pp. 268-269.

\(^{13}\) Ibid., p. 266.

\(^{14}\) It should be added that all curves of Fig. 2 have been calculated on the basis of actual diameter of driving wheels of the locomotive under test, 60.2 in. instead of nominal 62 in. The curves are, therefore, comparable with test data. In Fig. 1, however, the calculated curves were figured on the basis of nominal diameter of drivers (72 in.), while actually they were 69.6 in. If corrected, the curves of locomotive M-1 would come closer to test figures.
locomotive has, or has not, a feedwater heater. The effect of the feedwater heater has been included already in the moduli of 1932.

Thus, for all locomotives which have been so far analyzed, covering a very great range of heating surfaces and cylinder sizes, the soundness of the 1932 moduli with the 1934 modifications has been proved in practically all cases except only in these cases of stationary-plant tests (Figs. 1, 2, and 6), where the test figures should be higher than the moduli would indicate, and in one case of a road test (Fig. 9), where complete information is lacking.

**Formulas for 1932 and 1934 Curves**

A careful reader of the author's 1932 paper has undoubtedly discovered that the 1932 tractive-effort curves are very close to straight lines. This can be also verified by looking at the charts in the present paper. It is due to the fact that the author's moduli given in Table 1 follow the straight-line law in relation to speed. The values of $M_t$ for locomotives with feedwater heaters can be expressed very accurately by the following formula, with an error of not over 2.4 per cent:

$$M_t = 73.85 - 0.195n$$ \[13a\]

and without feedwater heaters by

$$M_t = 69.09 - 0.182n$$ \[13b\]

Formula \[13a\] can be replaced by an approximate formula in round figures

$$M_t = 75 - n/5$$ \[13c\]

Thus, the tractive-effort formula for locomotives with feedwater heaters will be

$$T_t = \frac{E_t}{D} (73.85 - 0.195n)$$ \[14\]

or approximately

$$T_t = \frac{E_t}{D} (75 - n/5)$$ \[14'\]

The percentages of modification are given in Fig. 5 by three lines, each consisting of a straight line and curve. As it can be seen from Fig. 5, the modifications can be approximately represented by straight lines intersecting the horizontal axis at $K = 15$. This has been shown in the figure by a dotted line in relation to the modification for $n = 50$. Similar straight lines could be drawn for the other two modification curves. On the basis of this approximation, the modification in per cent can be represented by formula

$$y = \left( \frac{30}{n + 10} - 0.15 \right) \frac{15 - K}{7}$$ \[15\]

in which $K$ is the known locomotive characteristic

$$K = \frac{E_t}{Vp_0}$$ \[10\]

The modified tractive effort $T_t'$ will thus be

$$T_t' = \frac{E_t}{D} (73.85 - 0.195n) (1 + y)$$ \[16\]

By the use of formulas \[10\], \[15\], and \[16\], the modified tractive effort can be calculated instead of being plotted by moduli and charts.

On the figures referred to representing charts for various locomotives, namely, Figs. 1–3 and 6–15, the points corresponding to formula \[14'\] have been marked by squares with dots in the center. It can be seen that they do not differ much from the performance curves and can be used with an accuracy sufficient for practical purposes. For instance, for the I-1s locomotive (Fig. 2), the discrepancy does not exceed 2.54 per cent if formula \[14\] is used, and +2.8 per cent if instead formula \[14'\] is preferred. In Table 5 a complete calculation by using moduli
and charts, and by using formula [14], is given for another locomotive, the Pennsylvania M-1. The error is not over 2.88 per cent. The results for other locomotives are similar.

### TABLE 5
(Pennsylvania M-1)

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<th>n</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
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<td>(M_p \times 1000)</td>
<td>65.6</td>
<td>4.4</td>
<td>42.7</td>
<td>34.1</td>
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<td>(T' = \frac{E_c}{D} M_p)</td>
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<td>37360</td>
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<td>21970</td>
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<tr>
<td>(T'(y \text{ from Fig. 5}))</td>
<td>61680</td>
<td>4590</td>
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<td>29790</td>
<td>21460</td>
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<tr>
<td>(y \text{ (from Eq. 16)})</td>
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<td>0.0495</td>
<td>0.0140</td>
<td>0.0</td>
<td>0</td>
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<td>Difference between values of (T') and (T'')</td>
<td>(-2.88)</td>
<td>(-1.16)</td>
<td>(+1.90)</td>
<td>(+2.20)</td>
<td>(-2.32)</td>
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Nevertheless, in the author's opinion, the above formulas, which have been derived here simply for the sake of completeness, should not be used when the moduli and charts are available, as they are more accurate results, but the formulas may be of use in many cases.

In using formulas [15] and [16] it should be remembered that no modification is required for locomotives with \(K = 14.26\) and larger, although in these cases a small correction may be necessary on the basis of formula [15]. It should be remembered that \(y = \pi\) for 200 and 250 rpm.

The 1932 moduli have been so far worked out for speeds between 50 and 250 rpm and intervals of 1 rpm, and the modifications were given for speeds between 4 and 150 rpm, and the same intervals. If these values for intermediate speeds with shorter intervals are desired, they can be easily worked out by plotting curves as functions of speed. (This connection see also footnote 3 and Eq. 4.) The formulas may have an advantage in that they give values for any speed.

### CONCLUSION

The 1932 moduli are applicable to, and do not require any changes for, modern locomotives for which \(K = 14.26\) and more, both for locomotives with and without feedwater heaters. For modern locomotives with \(K\) less than 14.26, modifications as stated above must be used for speeds between 50 and 150 rpm. The term "modern locomotive" is understood as defining a well-proportioned locomotive with a sufficiently large superheater, insuring at least 250 F superheat, proper valve motion, and the standard drafting arrangement, properly proportioned.

For some locomotives where forcing beyond what is considered reasonable limits of efficiency for performance is possible, from 10 to 15 per cent, or even higher tractive efforts, especially at higher speeds, are feasible. This depends upon the design of the locomotive as a whole, especially on such factors as the presence of combustion chamber, the depth and volume of firebox, the ratio between heating surface of firebox, tubes, and superheater, length of tubes and flues in relation to their diameter, steam distribution valves, and last but not least, the drafting arrangement.

On the basis of the experimental information available at present, it would be impossible to recommend formulas or curves embracing all these details, but it has been shown in the paper that no modern locomotive is giving a tractive-effort curve below what is recommended. The method, therefore, can be used as a reliable basis for numerical comparison of locomotives.