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A Graphical Solution of Windshield Heat Deicing Problems

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It is the purpose of this paper to present the problem behind the design of an air-heated double-paneled windshield for aircraft. A windshield ice-prevention nomograph is proposed as a rapid accurate method of making a design analysis of the air-heated windshield. With its use the designer has at his disposal a visual means of determining the relative importance of the variables included within the design, so as to arrive at the optimum solution for a given installation under the conditions imposed. The sample problems analyzed in this paper indicate that the required windshield strength be designed into the inner panel with an outer panel of tempered glass so as to obtain the highest outer-surface temperatures.

INTRODUCTION

UNTIL recently the usefulness of the airplane was somewhat limited due to the formation of ice on various surfaces. The early air-mail pilot had to turn back or try to fly through the icing weather, bailing out becoming necessary more times than not. Travelers became reluctant to fly in the winter months since the commercial air lines operated only in favorable weather. But the air lines had to maintain regular schedules and overweather flying developed. Even then many flight plans had to be changed in order to avoid icing conditions.

The war has changed this outlook. Military and naval operations must be made regardless of weather conditions. To overcome these difficulties the ice formations on aircraft must either be prevented or removed. Ice may be prevented by taking an ice-free path, although icing conditions might move in unexpectedly. Merely avoiding ice means that many scheduled flights and military missions would have to be canceled. Therefore the only logical way to maintain flight in icing weather would be to remove or prevent ice from forming on certain surfaces, which have given satisfactory service have been made on both mechanical, chemical, or thermal means.

The windshield is an example of such a surface since adequate vision must be maintained at all times for successful operation. A mechanical means of removing ice is the windshield wiper which has been used with little success. A chemical anti-icing fluid which reacts with the ice to prevent its adhesion to the glass surface was found to be satisfactory if proper distribution of the fluid over the surface could be maintained. With this type of installation, though, the visibility is impaired by clouding and smear resulting from the use of anti-icing fluids.

Heat is one of the most successful and practical means of preventing the formation of ice. One method is to circulate heated air against the inner surface of the windshield glass, raising the outer surface to a temperature above the freezing point of water. If ice has already formed, the surface under the ice is raised to a temperature of about 32 F which reduces the adhesion of ice to the glass surface; the air stream then carries the ice away. It is the design of an air-heated double-paneled windshield with which this paper deals exclusively.

AIR-HEATED DOUBLE-PANELED WINDSHIELD

Good vision through the windshield involves a threefold problem: (a) The prevention and removal of ice on the outer surface; (b) prevention and removal of frost or window fog from the inner surface; and (c) removal of rain from the outer surface. An air-heated installation capable of successfully preventing the formation of ice on the outer surface will remove both the frost and the rain. Several windshields of the double-paneled air-heated type have been constructed and tried repeatedly in icing conditions by the National Advisory Committee for Aeronautics. The device appears to be satisfactory and many installations which have given satisfactory service have been made on both commercial and military aircraft (1).2

A typical construction of an air-heated double-paneled windshield is shown in Fig. 1. The outer panel is laminated; a sheet of plastic, such as vinal, being inserted between two sheets of tempered glass. This laminated panel is needed for structural reasons only, for the windshield must pass satisfactorily an "impact test,"1 in order to demonstrate its ability to withstand collisions with birds during flight. The inner sheet completes the double-panel arrangement, being made from a sheet of plexiglas or its equivalent. The air gap thus formed allows the heated air to be concentrated against the outer panel, thereby improving the heat transfer over that experienced by a single-panel blast-air arrangement. The heated air enters the air gap through an inlet plenum which should be well designed to distribute the flow of air evenly over the windshield surface. The circulating-air exhaust may be either to the outside air or to the cockpit as shown, aiding the cabin-heating system.

The source of heat may be from heat exchangers, employing the waste heat in the engine exhaust, or from internal-combustion heaters located near the windshield. The source of pressure is usually the ram pressure of the airplane. However, a blower is often installed to supply the necessary pressure for defrosting during taxiing, take-off, or landing.

The design complications are now becoming evident. A limited amount of air will be forced through the air gap, the amount being determined by the gap length, gap thickness, and available pressure. A limit is also set on the air temperature allowed to enter the double panel, as there is a definite maximum temperature which the plastic can withstand without deterioration. Often the exhaust from the windshield enters the cockpit just over the pilot's head and to alleviate any discomfort due to this hot-air blast, a restriction is placed on the exhaust-air temperature. Finally, the windshield must be able to remove ice under the worst of flying conditions, which corresponds to the condition resulting in the highest coefficient of heat transfer over the outer surface. For a definite ambient air temperature and windshield size, the average heat-transfer factor for turbulent flow over a flat plate is given in reference (2), as varying directly

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1 Air Technical Service Command, A.A.F.; formerly Air Conditioning Design Engineer, Douglas Aircraft Company, Inc.
2 Numbers in parentheses refer to the Bibliography at the end of the paper.
with the 0.8 power of the air velocity and density. Expressed in

\[ (Average) \ h_0 \propto (V \gamma)^{0.8} \]

It is therefore apparent that the most heat is required at the
lowest altitude and the maximum airplane speed.

A moment of reflection will indicate a definite need for a graphi­
cal solution to such a design problem. An identical calculation
would be required for each proposed structural change. Then
for each air flow there is a corresponding gap size which may or
may not be a practical size for ease of construction. In time
of war there is also a real need for speed in designing. On the
other hand, a graphical solution by the use of a nomograph al­
lows a visual means of determining the effect of each variable
so as to arrive at the optimum design for a particular installa­
tion.

Bearing these factors in mind, the windshield-ice-prevention
nomograph, Fig. 2, was prepared. The following nomenclature
corresponds to the symbols used in the development of the equa­
tions appearing on the nomograph:

**NOMENCLATURE**

- \( A \) = cross-sectional area of gap, sq ft
- \( b \) = windshield width, ft
- \( C_p \) = specific heat of air at constant pressure, Btu/lb deg F
- \( d \) = gap thickness, in.
- \( D_e \) = equivalent diameter of air passage between panels, ft
- \( f \) = friction factor for air flow, dimensionless
- \( g \) = acceleration of gravity, \( 4.17 \times 10^8 \) ft per hr per hr
- \( G \) = weight flow of air per unit area, lb/(hr)(sq ft)
- \( h \) = average convection heat-transfer coefficient on cockpit
  side of inner panel, Btu/(hr)(sq ft) (deg F)
- \( h_i \) = average convection heat-transfer coefficient inside air
  gap, Btu/(hr)(sq ft) (deg F)
- \( h_o \) = average convection heat-transfer coefficient over
  outer windshield surface, Btu/(hr)(sq ft) (deg F)
- \( k \) = thermal conductivity of each lamination, Btu/(hr)
  (sq ft) (deg F)/in.
- \( L \) = length of windshield parallel to direction of air flow in
  gap, in.
- \( l \) = thickness of each lamination, in.
- \( \Delta P \) = total pressure drop of air in gap, in. water
- \( \Delta P' \) = pressure drop of air in gap per foot of windshield length
  at sea level, in. water
- \( P \) = perimeter of gap, ft
- \( q_i \) = heat transferred through inner panel, Btu/(hr)(sq ft)
- \( q_o \) = heat transferred through outer panel, Btu/(hr)(sq ft)
- \( R \) = Reynolds number, dimensionless
- \( R_i \) = thermal resistance of inner panel, (deg F)(sq ft)/Btu
  /hr
- \( R_o \) = thermal resistance of outer panel, (deg F)(sq ft)/Btu
  /hr
- \( T \) = average temperature of air in gap, deg F
- \( \Delta T \) = temperature drop of circulating air in gap, deg F
- \( \Delta T_{av} \) = arithmetic temperature difference between average
  temperature of air in gap and outside air, deg F
- \( \Delta T_{av} \) = arithmetic temperature difference between average
  temperature of air in gap and cockpit temperature, deg F
- \( t_1 \) = entering circulating air temperature, deg F
- \( t_2 \) = leaving circulating air temperature, deg F
- \( U_o \) = over-all heat-transfer coefficient across outer panel,
  Btu/(hr)(sq ft) (deg F)
- \( U_i \) = over-all heat-transfer coefficient across inner panel,
  Btu/(hr)(sq ft) (deg F)
- \( v_1 \) = specific volume of air at entrance and exit, respectively,
  of a duct, cu ft per lb
- \( W \) = total air flow to windshield, lb per hr
- \( w \) = air flow through windshield per foot of windshield
  width, lb per hr per ft
- \( \gamma \) = weight density of air, lb per cu ft
- \( \gamma_{av} \) = average weight density of air in a duct, lb per cu ft
- \( \mu \) = absolute viscosity of air, lb/(hr)(ft.)

**THERMODYNAMIC ANALYSIS**

An analysis of the thermodynamic design of an air-heated
double-paneled windshield will now be made. The final equa­
tions derived during the course of the following analysis are
found in nomographic form in Fig. 2.

Referring to Fig. 1, if the temperature rise of the outer sur­
face over the outside air be denoted as \( \Delta T_o \), and if the average
heat-transfer coefficient over this surface be denoted as \( h_o \),
then the heat transferred from this surface to the outside air per
square foot of surface is given by (3)

\[ q_o = h_o \Delta T_o \]
The heat transferred through the outer panel is identical with that as given in Equation [1]; therefore

\[ q_e = \frac{U_s}{h_i} + K_a + \frac{1}{h_o} \]  \hspace{1cm} [3]

This is Newton's law of heat transfer through a solid wall from one fluid to another. In Equation [2] the arithmetic-mean temperature difference is used in place of the more correct logarithmic-mean temperature difference, introducing an error of less than 4 per cent (4), as will be explained later in the sample problem. The over-all heat-transfer coefficient \( U_o \) is defined as

\[ \frac{1}{U_o} = \frac{1}{h_i} + K_a + \frac{1}{h_o} \]  \hspace{1cm} [3]

The thermal resistance to heat transfer, \( R_a \), for a laminated panel depends on the thickness \( l_i \) and thermal conductivity \( k_i \), of each lamination, as

\[ R_a = \frac{l_i}{k_i} + \frac{l_2}{k_2} + \ldots + \frac{l_n}{k_n} \]  \hspace{1cm} [4]

The heat-transfer coefficient inside the gap formed by the two panels is calculated (5) by the equation

\[ h_i = 5.56 \times 10^{-4} \frac{T_0^{0.344} (G)^{0.80}}{D_B^{0.19}} \]  \hspace{1cm} [5]

Although Equation [5] was developed for flow in pipes, it can be applied with reasonable accuracy to small rectangular ducts by the substitution of the equivalent diameter \( D_B \), for the pipe diameter \( D \). Letting \( W = \frac{A}{b} \), the air flow through the gap per foot of windshield width; then

\[ G = \frac{W}{A} = \frac{W}{d b} = \frac{12 W}{12} \]  \hspace{1cm} [6]

By definition the equivalent diameter for a rectangular duct is

\[ D_B = \frac{4 A}{P} = \frac{4 \times \frac{d}{12} \times b}{2d + 2b} \]  \hspace{1cm} [7]

Since the gap width is very much smaller than the gap length, slight error will be introduced in dropping the \( \frac{2d}{12} \) term from the denominator of the fraction in Equation [7]; therefore

\[ D_B = \frac{6 d}{12} \]  \hspace{1cm} [8]


\[ h_i = 5.56 \times 10^{-4} T_0^{0.344} \left( \frac{12 W}{6} \right)^{0.80} \]  \hspace{1cm} [9]

or

\[ h_i = 58.1 \times 10^{-4} T_0^{0.344} \left( \frac{2W}{6} \right)^{0.80} \]  \hspace{1cm} [10]

The well-known Fanning equation for pressure drop in inches of water for a straight duct (3) is

\[ \Delta P' = 4f \frac{L}{D_B} \frac{G^2}{2g} \gamma 5.2 \]  \hspace{1cm} [11]

Since the change in specific volume of the air as it passes through the windshield is negligible, the second term in the right side of Equation [11] may be omitted. Therefore

\[ \Delta P' = 4f \frac{L}{D_B} \frac{G^2}{2g} \gamma 5.2 \]  \hspace{1cm} [12]

Denoting the pressure drop per foot of windshield length as \( \Delta P' \)

\[ \Delta P' = 4f \frac{L}{D_B} \frac{1}{2g} \frac{1}{\gamma} \frac{5.2}{6} \]  \hspace{1cm} [13]

The friction factor \( f \) is given (3) by

\[ f = \frac{0.046}{R_{e,20}} \]  \hspace{1cm} [14]

where

\[ R = \frac{12 \nu d}{d} \]  \hspace{1cm} [15]

The values of \( \gamma \) and \( \nu \) will be evaluated for sea-level altitude and at an average temperature of 200 deg F, which is the approximate average air temperature in the gap for the majority of the practical windshield designs (6). Therefore

\[ \gamma = 0.060 \text{ lb per cu ft, and } \nu = 0.053 \text{ lb/(hr)(ft)} \]

Substituting the value of \( \nu \) into Equation [15], evaluating the friction factor \( f \), and then replacing \( f, \gamma, \) and \( g \) by their numerical values in Equation [13], this equation becomes

\[ \Delta P' = 29.65 \times 10^{-4} \frac{G^2}{g} \frac{d}{\nu} \]  \hspace{1cm} [16]

Note that Equation [16] was developed for sea-level altitude and for a windshield length of 1 ft. It will be necessary to correct the chart value of \( \Delta P' \) for the design altitude and windshield length under consideration.

It is now necessary to determine the amount of heat transferred into the cockpit through the inner panel, for the total temperature drop in the air passing through the gap depends on the total amount of heat transferred from the double panel. The over-all coefficient of heat transfer is calculated from the equation

\[ \frac{1}{U_i} = \frac{1}{h_i} + K_a + \frac{1}{h_o} \]  \hspace{1cm} [17]

The assumption is made that the convection-heat-transfer coefficient, \( h \), on the cockpit side of the inner panel is equal to 1.65 Btu/(hr)(sq ft) (deg F) (7). Then the heat transferred through the inner panel from the air in the gap to the cockpit is

\[ q_i = U_i \Delta t \]  \hspace{1cm} [18]
NOTE:
1. SAMPLE PROBLEM IS FOR COLUMN 1 OF TABLE
2. DOTTED LINES INDICATE A TRANSFER OF A VALUE FROM ONE SCALE TO ANOTHER
3. ARROWS MARKED THIS INDICATE STEPS OF SOLUTION

FIG. 2 Windshield Ice-Prevention Nomograph
It would be well to note that any additional surface temperature rise due to the phenomena of kinetic heating has been neglected in this analysis. Kinetic heating has been neglected for three reasons; (a) for speed of best range kinetic heating is a minimum; (b) the temperature rise depends on the specific heat of "wet" air, the value of which is not accurately known since the exact amount of water present in the air under icing conditions is unknown; and (c) the calculated value of the surface temperature by the nomograph will be conservative.

Applying the Nomograph

In order to illustrate the use of the nomograph, a sample problem will be calculated by use of the eight equations just given and compared to a nomographic solution. However, before the design equations can be applied, certain data relating to materials and dimensions of the windshield, entering-air temperature, leaving-air temperature, air speed, and allowable pressure drop must be known or assumed. For a typical case these data are as follows:

1. For the outer panel (refer to Fig. 1)
   \[ l_1 = 0.125 \text{ in., } k_1 = 7 \text{ Btu/(hr)(sq ft)(deg F)/in.} \] (8)
   \[ t_1 = 0.020 \text{ in., } k_2 = 1.44 \text{ Btu/(hr)(sq ft)(deg F)/in.} \] (8)
2. For the inner panel (refer to Fig. 1)
   \[ l = 0.250 \text{ in., } k = 1.45 \text{ Btu/hr sq ft deg F/in.} \] (8)
3. For the windshield dimensions
   \[ L = 18 \text{ in.} \]
   \[ b = 13 \text{ in.} \]
4. Entering-air temperature, \( t_e \leq 230 \text{ deg F} \)
5. Leaving-air temperature, \( t_l \leq 170 \text{ deg F} \)
6. For 25,000 ft altitude, minimum air speed is such that
   \[ h_0 = 25 \text{ Btu/(hr)(sq ft) (deg F)}; \text{ total allowable } \Delta P' = 10 \text{ in. water} \]
7. For sea level, maximum air speed is such that
   \[ h_0 = 40 \text{ Btu/(hr)(sq ft)(deg F)}; \text{ total allowable } \Delta P' = 10 \text{ in. water} \]
8. Outside-air temperature = 0 deg F.
9. Outer-surface temperature to be maintained = 40 deg F
10. Cockpit temperature = 70 deg F

The problem then resolves itself into finding what value of \( d \) in conjunction with \( w \) will provide the necessary surface temperature within the allowable pressure drop at 25,000 ft. At this altitude and at minimum speed the ram pressure is the smallest. Then with the gap size set, the maximum required heat input is determined from the maximum speed at sea level.

The solution of this problem proceeds in the following manner:
From Equation [1] at 25,000 ft
\[ q_0 = 25 \times 40 = 1000 \text{ Btu/hr sq ft} \]
Assume an average temperature of circulating air in gap of 200 F. Then from Equation [2]
\[ U_0 = \frac{1000}{(200 - 0)} = 5 \text{ Btu/(hr)(sq ft)(deg F)} \]
From Equation [4]
\[ R_0 = \frac{0.125}{7} + \frac{0.020}{1.44} + \frac{0.125}{7} = 0.0496 \text{ (deg F)/(sq ft)/Btu/hr) } \]
Therefore from Equation [3]
\[ \frac{1}{h_1} = \frac{1}{5} - 0.0496 - \frac{1}{25} = 0.1104 \]
\[ h_1 = 9.05 \text{ Btu/(hr)(sq ft)(deg F)} \]
Now, evaluating Equation [10], for \( d = \frac{1}{4} \text{ in. (assumption), and solving for } w \)
\[ T_{0.298} = \left(460 + 200 \right)^{0.298} = 6.81 \]
\[ w^{0.80} = \frac{0.125 \times 9.05 \times 10^4}{58.1 \times 6.81} = 28.6 \]
\[ w = 66 \text{ lb per hr per ft} \]

Total air flow to the windshield is
\[ W = 66 \times 13 = 71.5 \text{ lb per hr} \]

Now from Equation [16]
\[ \Delta P' = 29.65 \times 10^{-\frac{1}{3}} \left(\frac{66}{0.125}\right)^{1.8} = 0.289 \text{ in. water for sea-level altitude} \]
For 25,000 ft altitude, the total pressure drop is
\[ \Delta P = \Delta P' \left(\frac{29.92}{11.1}\right) \]
where 29.92 = standard atmospheric pressure at sea level, in. Hg; and 11.1 = standard atmospheric pressure at 25,000 ft, in. Hg.

Solving
\[ \Delta P = 0.289 \times 2.69 \times 1.5 = 1.165 \text{ in. water} \]
which is less than that available.

Now \[ R_i = \frac{0.250}{1.45} = 0.1725 \text{ (deg F)/(sq ft)/Btu/hr} \]
Table 1: Data for Sample Problem Solution from Nomograph, Fig. 2

<table>
<thead>
<tr>
<th>Column No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<tr>
<td>Altitude</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>S.L.</td>
<td>S.L.</td>
<td>S.L.</td>
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<td>Outside Air Temperature, °F</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Outside Average Heat Transfer Coefficient, ( h_0 )</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Surface Temperature To Be Maintained, °F</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
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</tr>
<tr>
<td>Heat Transferred Through Outer Panel, ( q_0 )</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1600</td>
<td>1600</td>
<td>1600</td>
</tr>
<tr>
<td>Average Temperature Of Circulating Air In Gap, ( t_{av} ), °F</td>
<td>200</td>
<td>185</td>
<td>190</td>
<td>180</td>
<td>200</td>
<td>200</td>
<td>205</td>
<td>210</td>
</tr>
<tr>
<td>Over-all Coefficient Of Heat Transfer, ( U_0 )</td>
<td>5.0</td>
<td>5.4</td>
<td>5.26</td>
<td>5.55</td>
<td>5.0</td>
<td>8.0</td>
<td>7.80</td>
<td>7.63</td>
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<td>Thermal Resistance Of Outer Panel, ( R_0 )</td>
<td>0.0496</td>
<td>0.0496</td>
<td>0.0496</td>
<td>0.0496</td>
<td>0.0496</td>
<td>0.0496</td>
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<tr>
<td>Inside Convection Heat Transfer Coefficient In Gap, ( h_1 )</td>
<td>9.05</td>
<td>10.6</td>
<td>10</td>
<td>11.0</td>
<td>9.05</td>
<td>20.5</td>
<td>19.0</td>
<td>17.6</td>
</tr>
<tr>
<td>Air Flow Per Foot, ( W' )</td>
<td>66</td>
<td>82</td>
<td>100</td>
<td>60</td>
<td>110</td>
<td>185</td>
<td>220</td>
<td>250</td>
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<tr>
<td>Pressure Drop Per Foot, ( \Delta P' )</td>
<td>0.29</td>
<td>0.42</td>
<td>0.35</td>
<td>0.60</td>
<td>0.23</td>
<td>1.90</td>
<td>1.30</td>
<td>0.96</td>
</tr>
<tr>
<td>Total Pressure Drop At The Altitude Considered, ( \Delta P' )</td>
<td>1.17</td>
<td>1.70</td>
<td>1.42</td>
<td>2.43</td>
<td>0.927</td>
<td>2.85</td>
<td>1.95</td>
<td>1.64</td>
</tr>
<tr>
<td>Thermal Resistance Of Inner Panel, ( R_1 )</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
</tr>
<tr>
<td>Over-all Coefficient Of Heat Transfer, ( U_1 )</td>
<td>1.12</td>
<td>1.14</td>
<td>1.13</td>
<td>1.145</td>
<td>1.126</td>
<td>1.20</td>
<td>1.193</td>
<td>1.190</td>
</tr>
<tr>
<td>Heat Transferred Through Inner Panel, ( q_1 )</td>
<td>145</td>
<td>132</td>
<td>135</td>
<td>125</td>
<td>145</td>
<td>157</td>
<td>162</td>
<td>170</td>
</tr>
<tr>
<td>Total Temperature Drop In Circulating Air, ( \Delta t ), °F</td>
<td>108</td>
<td>88</td>
<td>70</td>
<td>118</td>
<td>68</td>
<td>60</td>
<td>50</td>
<td>44</td>
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<tr>
<td>Entering Air Temperature, ( t_1 ), °F</td>
<td>254</td>
<td>229</td>
<td>225</td>
<td>239</td>
<td>234</td>
<td>230</td>
<td>230</td>
<td>232</td>
</tr>
<tr>
<td>Leaving Air Temperature, ( t_2 ), °F</td>
<td>146</td>
<td>141</td>
<td>135</td>
<td>121</td>
<td>166</td>
<td>170</td>
<td>180</td>
<td>188</td>
</tr>
<tr>
<td>Heater Requirement Assuming a 250 °F Rise</td>
<td>12,000</td>
<td>11,500</td>
<td>11,600</td>
<td>12,000</td>
<td>11,500</td>
<td>11,500</td>
<td>11,500</td>
<td>11,500</td>
</tr>
</tbody>
</table>

And from Equation [17]

\[
\frac{1}{U_t} = \frac{1}{9.05} + 0.1725 + \frac{1}{1.65} = 0.8889
\]

\[ U_t = 1.125 \text{ Btu/(hr)(sq ft)(deg F)} \]

Then by Equation [18], for a cockpit temperature of 70 °F

\[ q_t = 1.125 \times (200 - 70) \]

\[ q_t = 146 \text{ Btu/(hr)(sq ft)} \]

The total temperature drop in the gap is, from Equation [19]

\[ \Delta t = \frac{(1000 + 146)(1.50)}{66 	imes 0.24} = 108.5 \text{ deg F} \]

Hence the entering-air temperature must be

\[ t_1 = 200 + \frac{108.5}{2} = 254.2 \text{ deg F} \]

and

\[ t_2 = 200 - \frac{108.5}{2} = 145.7 \text{ deg F} \]

It is noticed that the entering-air temperature \( t_1 \) is higher than the maximum allowed. Therefore the complete problem must be recalculated, assuming a lower average gap air temperature until all conditions are satisfied.

The solution for this same problem is shown in Fig. 2 and tabulated in Table 1, column 1. Good agreement is noticed when the chart values are compared with the calculated values.

A simple calculation will show the validity of using the arithmetic-mean rather than the logarithmic-mean temperature difference. Referring to Fig. 3 the temperature difference (\( \Delta t_1 \)) between the entering-air temperature and outside-air temperature is 254 deg F, and for the exit condition, \( \Delta t_2 = 146 \text{ deg F} \). From refer-
The arithmetic-mean temperature difference would be
\[
\Delta t_{av} = \frac{254 - 146}{2} = 200 \text{ deg F}
\]

Error = 2\% per cent.

Table 1 has been prepared by the use of the nomograph, Fig. 2, to illustrate one method of determining the correct gap size and air flows. This method is to assume various gap sizes and, in conjunction with the desired outer-surface temperature, solve for the required air flows and circulating-air temperatures.

Column 2 is a corrected version of column 1, indicating that the average temperature of the circulating air should be 185 deg F, instead of 200 deg F in order that the design conditions be satisfied. Other gap sizes were then assumed as indicated in columns 3, 4, and 5, and the problem again solved, the results indicating that, at 25,000 ft, a \( \frac{1}{8} \) in. gap is out of the question due to the high pressure drop but that a \( \frac{1}{32} \) or \( \frac{1}{16} \) in. gap may be satisfactory. Columns 6, 7, and 8 prove that at sea level a \( \frac{1}{8} \) in. gap is more to be desired than either of the larger gaps because of the lower heat requirement and lower exit-air temperature from the windshield.

Another method of solving the sample problem, by use of the nomograph, Fig. 2, is to determine the maximum air flows for various gap sizes from the available pressure and to solve for the resulting outer-surface temperature, choosing the gap size which satisfies the design conditions. By this method Table 2 was prepared. Referring to the design requirements of the preceding sample problem, the pressure available at an altitude of 25,000 ft is 1.85 in. of water. This corresponds to a pressure drop at sea level per foot of windshield length of

\[
\Delta P' = 1.85 \left( \frac{20.92}{18} \right) = 0.457 \text{ in. water}
\]

This pressure drop determines the air flow through the windshield which, in turn, determines the outer-surface temperature. In Table 2, column 1 shows that a \( \frac{1}{32} \) in. gap does not give the required 40 deg F surface temperature, but that this temperature rise is realized by a \( \frac{1}{8} \), \( \frac{5}{32} \), or \( \frac{3}{16} \) in. gap size, as shown by columns 2, 3, and 4.

If the air flow through the windshield were determined by the comparatively large amount of pressure available under sea-level flying conditions, an oversized heating system would result.

### Table 2: DATA FOR ALTERNATIVE SOLUTION OF PROBLEM

<table>
<thead>
<tr>
<th>Column No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
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<tbody>
<tr>
<td>Altitude</td>
<td>25000</td>
<td>25000</td>
<td>25000</td>
<td>25000</td>
<td>S.L.</td>
<td>S.L.</td>
<td>S.L.</td>
<td>S.L.</td>
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<tr>
<td>Outside Air Temperature, °F</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Pressure Drop At Altitude Considered, ( \Delta P )</td>
<td>1.85</td>
<td>1.85</td>
<td>1.85</td>
<td>1.85</td>
<td>2.85</td>
<td>1.95</td>
<td>1.44</td>
<td>2.85</td>
</tr>
<tr>
<td>Pressure Drop Per Foot Corrected To Sea Level, ( \Delta P' )</td>
<td>.457</td>
<td>.457</td>
<td>.457</td>
<td>.457</td>
<td>1.90</td>
<td>1.30</td>
<td>.96</td>
<td>1.90</td>
</tr>
<tr>
<td>Air Flow, ( \phi )</td>
<td>51</td>
<td>86</td>
<td>120</td>
<td>163</td>
<td>185</td>
<td>220</td>
<td>250</td>
<td>185</td>
</tr>
<tr>
<td>Average Temperature Of Circulating Air In Gap, ( t_{av} ), °F</td>
<td>175</td>
<td>185</td>
<td>190</td>
<td>200</td>
<td>200</td>
<td>195</td>
<td>210</td>
<td>195</td>
</tr>
<tr>
<td>Inside Convection Heat Transfer Coefficient In Gap, ( h_{in} )</td>
<td>9.9</td>
<td>11.0</td>
<td>11.4</td>
<td>12.2</td>
<td>20.5</td>
<td>19.0</td>
<td>17.6</td>
<td>20.45</td>
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<tr>
<td>Thermal Resistance Of Outer Panel, ( R_t )</td>
<td>.0496</td>
<td>.0496</td>
<td>.0496</td>
<td>.0496</td>
<td>.0496</td>
<td>.0496</td>
<td>.0496</td>
<td>.0357</td>
</tr>
<tr>
<td>Over-all Coefficient Of Heat Transfer - Outer Panel, ( h_{o} )</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
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<tr>
<td>Heat Transferred Through Outer Panel, ( q_{o} )</td>
<td>910</td>
<td>1040</td>
<td>1075</td>
<td>1160</td>
<td>1160</td>
<td>1160</td>
<td>1160</td>
<td>1775</td>
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<tr>
<td>Surface Temperature, °F</td>
<td>36.4</td>
<td>41.6</td>
<td>43</td>
<td>46.4</td>
<td>40</td>
<td>40</td>
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<td>44.5</td>
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<td>Thermal Resistance Of Inner Panel, ( R_i )</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>1.725</td>
<td>.0496</td>
</tr>
<tr>
<td>Over-all Coefficient Of Heat Transfer, Inner Panel, ( h_{i} )</td>
<td>1.13</td>
<td>1.145</td>
<td>1.148</td>
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<td>1.20</td>
<td>1.193</td>
<td>1.19</td>
<td>1.41</td>
</tr>
<tr>
<td>Heat Transferred Through Inner Panel, ( q_{i} )</td>
<td>116</td>
<td>131</td>
<td>136</td>
<td>150</td>
<td>157</td>
<td>162</td>
<td>170</td>
<td>180</td>
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<tr>
<td>Temperature Drop In Circulating Air, ( \Delta t ), °F</td>
<td>118</td>
<td>88</td>
<td>64</td>
<td>50</td>
<td>50</td>
<td>44</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Entering Air Temperature, ( t_{1} ), °F</td>
<td>234</td>
<td>225</td>
<td>222</td>
<td>225</td>
<td>230</td>
<td>230</td>
<td>232</td>
<td>230</td>
</tr>
<tr>
<td>Leaving Air Temperature, ( t_{2} ), °F</td>
<td>116</td>
<td>141</td>
<td>158</td>
<td>175</td>
<td>170</td>
<td>180</td>
<td>188</td>
<td>160</td>
</tr>
<tr>
<td>Heater Requirement Assuming a 250°F Rise</td>
<td>12,000</td>
<td>14,300</td>
<td>16,200</td>
<td>12,000</td>
<td>14,300</td>
<td>16,200</td>
<td>12,000</td>
<td>14,300</td>
</tr>
</tbody>
</table>

### Footnotes:
1. \( \Delta t_{LM} = \frac{\Delta t_1 - \Delta t_2}{\log_{10} \Delta t_1 - \log_{10} \Delta t_2} \)
2. \( \Delta t_{av} = \frac{254 - 146}{2} = 200 \text{ deg F} \)
Hence the maximum required heater capacity should be based on an adequate outer-surface temperature and the gap size as determined by the minimum available pressure at the critical altitude (25,000 ft in this sample problem). A manual or automatic temperature control of the heater-outlet temperature must be provided by utilizing, through a valve or damper, the excess ram pressure at the low altitudes. Thus the maximum heat requirement was determined by method 1 as tabulated in Table 1. Columns 5, 6, and 7 of Table 2 were merely copied from columns 6, 7, and 8 of Table 1.

Another windshield configuration worthy of consideration is to design the required impact strength into the inner windshield panel and to construct the outer panel of one sheet of tempered glass. By this arrangement a higher outer-surface temperature can be maintained due to the decreased thermal resistance of the outer panel. As an example of such a design the sample problem was again solved by the windshield ice-prevention nomograph, assuming that the laminated panel replaces the plexiglas inner panel, and that the outer panel is constructed from a sheet of 1/4-in. tempered glass. Referring to column 8 of Table 2, a 4.5-deg F higher surface temperature rise was obtained using the same 1/4-in. gap size and air flow, with a further desired reduction in the leaving-air temperature of the circulating air.

**Conclusion**

The paper demonstrates that a logical method of solution for windshield heat deicing problems may be derived from a nomograph. One of its most important features is the saving of time in making a detailed design analysis of such an installation. As quick as two points can be connected with a straightedge the equation is solved with the position of the decimal point already ascertained. Of course a knowledge of the design equations is necessary for a complete understanding of this type of problem, but once the background study is completed, a graphical solution is required to alleviate the monotony of making the same calculation over and over for each proposal.

Inasmuch as the exact heat requirement needed to keep the windshield outer surface at a specified temperature in icing conditions is not accurately known, an installation designed without the benefit of kinetic heating should prove successful in adverse weather. Also, the possibility of constructing the outer panel of tempered glass and designing the required structural strength into the inner panel should not be overlooked because of the increased heat-transmission quality of this type of outer panel. The smallest gap size possible within the limits of the problem is to be desired, as the required heater size and exit-air temperature are kept to a minimum when the windshield is designed as the most efficient heat exchanger possible.

It may therefore be seen that an air-heated windshield is more than just two panes of glass, the gap of which is assumed and to which only the excess air from the cockpit heating system is allowed to be utilized. It is hoped that his windshield ice-prevention nomograph will be of some benefit to the aircraft engineer as a timesaver.

**BIBLIOGRAPHY**

An Acceleration Damper: Development, Design, and Some Applications

By Paul Lieber and D. P. Jensen

An acceleration damper is essentially an impact damper, consisting of a mass particle within a container such that the particle has specified freedom to move relative to the container. The efficiency of the damper depends critically on the freedom of the particle relative to its container. The energy of the mass particle, which is energized by its container, is dissipated in impact. The mechanism and theory of the acceleration damper are discussed and developed in this paper. The theory yields formulas from which the acceleration damper can be designed efficiently for specific application. Theory and procedure for calculating the motion of a mechanical system, as influenced by a given acceleration damper, are developed. Tests were conducted and the agreement between theory and experiment is found to be good. These results show that friction forces acting on the mass particle are detrimental to the efficiency of the damper. Finally, a method for calculating the effect of the acceleration damper on flutter is also developed, which is accompanied by numerical results related to an actual airplane. It is indicated that the acceleration damper may have fundamental and numerous applications, some of which are fatigue, helicopter vibration control, aircraft vibration control in general, and to facilitate flight testing for flutter without endangering aircraft.

Nomenclature

The following nomenclature is used in the paper:

- $\theta$ = phase angle between damped system and mass particle
- $x$ = rectilinear co-ordinate of mechanical system, ft
- $s$ = curvilinear co-ordinate of mechanical system, ft
- $x_0$ = maximum rectilinear amplitude of system, undergoing simple harmonic motion, ft
- $d$ = free path of mass particle in its container, ft
- $K$ = spring constant of schematic mechanical system, lb per ft
- $M$ = mass of schematic mechanical system in slugs
- $m$ = mass of free particle of acceleration damper in slugs
- $t$ = time variable, sec
- $\omega$ = frequency of oscillation, radians per sec
- $F_p(t)$ = force exerted on vibrating system by mass particle
- $s_0$ = maximum curvilinear amplitude of system, ft
- $f(u(x))$ = unit step function whose periodicity is a function of $\theta$

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

The following applications are indicated and warrant experimental investigation:

1. Facilitate flight testing for flutter without endangering aircraft
2. Fatigue control of primary and secondary structures
3. Reduction of vibration amplitude associated with buffeting
4. Control of vibration effects associated with compressibility
5. Control of helicopter vibration
6. General vibration control.

One of the useful features of this damper is that from its very nature it can be installed to provide a system of uniformly dis-
Distributed damping forces and thereby have little effect on the modes of oscillation themselves. Since the damping force is a function of absolute acceleration its installation is simple.

This paper considers some theoretical aspects of the acceleration damper itself and gives a formula enabling one to use empirical damping data in theoretical investigations of flutter and vibration phenomena. The elastic rebound between the mass particle and its container is assumed to be zero.

Fig. 1

**Theoretical Aspects of Acceleration Dampers**

Consider an acceleration damper installed on an inertia elastic system having one degree of freedom, being forced periodically by $F(t)$, such that a steady-state simple harmonic motion is maintained. It is imagined that a power supply $P(t)$ exists, necessary to maintain such motion. The work done by the mass particle on the $M$ and $K$ system in a complete cycle is given by the integral

$$\int_0^{2\pi/\omega} F_p(t) \cdot \dot{x} \, dt = \int_0^{2\pi/\omega} F_p(t) \cdot \dot{x} \, dt$$  \[1\]

where

- $F_p(t)$ = force exerted on vibrating system by mass particle $m$ when in contact with $M$
- $\omega$ = frequency of oscillation, radians per sec
- $x$ = position of system
- $x_0$ = maximum amplitude of system
- $t$ = time variable
- $M$ = mass of system
- $K$ = spring constant of system
- $F_p(t)$ = periodic exciting force

Since $M$ undergoes simple harmonic motion the terms $F_p(t)$ and $x$ of the integral in Equation [1] are illustrated graphically in Fig. 2.

![Graph showing $F_p(t)$ and $x$](image)

It is clear that due to the discontinuities in the function $F_p(t)$ the integral in Equation [1] is finite, i.e., $F_p(t)$ is dissipative. In maintaining a constant amplitude of oscillation it is therefore necessary to provide a constant and finite power input $P(t)$ over a cycle.

Since, as will be shown, the effective damping coefficient of an acceleration damper is completely determined for a constant amplitude $x_0$ by the phase angle $\theta$ between the motion of the mass particle $m$ and the mass $M$, the following analysis is based on steady-state simple harmonic motion. This analysis will bring out the nature of the dissipative forces as well as provide a formula for calculating the efficiency of the damper as a function of the phase angle $\theta$.

Consider the work done by the system of forces acting on the mass $M$ over one cycle

$$\int_0^{2\pi/\omega} Kx \ddot{x} \, dt + \int_0^{2\pi/\omega} M\ddot{x} \, dt + \int_0^{2\pi/\omega} m\ddot{x}f(\omega t) \, dt + \sum R$$

$$\int_0^{2\pi/\omega} m(x_{0\omega} - \dot{x})\delta(t - RT)\dot{x} \, dt = \text{work done per cycle}$$  \[2\]

For a brief discussion of the Dirac function refer to work by Carslaw and Jaeger. The effect of the acceleration damper is represented in two integrals in Equation [2]. The integral $\int_0^{2\pi/\omega} m\ddot{x}f(\omega t) \, dt$ is the work done by the particle on $M$ after exerting its impact. When the particle is in contact with $M$, $f(\omega t) = -1$, and when particle is free, $f(\omega t) = 0$. The work exerted by the particle on $M$ upon establishing contact with it is represented by the integral

$$\int_0^{2\pi/\omega} m(x_{0\omega} - \dot{x})\delta(t - RT)\dot{x} \, dt$$

if the motion is simple harmonic, then

$$x = x_0 \sin \omega t$$

$$\ddot{x} = -\omega^2x_0 \sin \omega t$$


$$K \int_0^{2\pi/\omega} x_0 \sin \omega t \cdot x_0 \cos \omega t dt + M \int_0^{2\pi/\omega} \omega^2x_0 \sin \omega t \cdot \omega x_0 \cos \omega t dt + M$$

$$\int_0^{2\pi/\omega} \omega^2x_0 \sin \omega t \cdot \omega x_0 \cos \omega t dt = \frac{2\pi}{W}$$  \[4\]

work done per cycle.

The first two integrals in Equation [4] vanish and the work done is determined by the remaining integrals whose values depend only on the design and efficiency of the acceleration damper. Equation [4] reduces to

$$-m\omega^2x_0^2 \int_0^{2\pi/\omega} \sin \omega t \cos \omega t f(\omega t) dt + m\omega^3x_0^2$$

$$\int_0^{2\pi/\omega} (1 - \cos \omega t) \delta(t - RT) \cos \omega t dt = \frac{2\pi}{W}$$  \[5\]

The first integral can be readily evaluated for a given phase angle $\theta$ since the phase angle completely defines the function $f(\omega t)$ which is illustrated in Fig. 3.

By integrating $\int_0^{2\pi/\omega} \sin \omega t \cos \omega t f(\omega t) \, dt$ for a time interval in which $f(\omega t)$ is unity, and which is defined by $\theta$, the contribution of the term $m\omega^2x_0^2 \sin \omega t \cos \omega t f(\omega t) \, dt$ to $\frac{2\pi}{W}$ can be readily calculated. It is

$$+ m\omega^2x_0^2 \frac{2\pi}{W} \frac{1}{2} \sin^2 \theta$$

$$= -m\omega^2x_0^2 \sin^2 \theta \theta$$  \[6\]

Periodicity of Contact

By periodicity of contact is meant the time elapsed between successive contacts of the particle \( m \) with \( M \). As a preliminary to calculating the periodicity of contact, consider the time elapsed in which the mass particle \( m \) leaves \( M \) and re-establishes contact with \( M \); (if the motion is simple harmonic, i.e., \( x = x_0 \sin \omega t \)) it is

\[
T_{n+1} = \frac{d + \int_{n\omega}^{T_{n+1}} x dt}{x_{\infty}} = \frac{d + x_0 \int_{n\omega}^{T_{n+1}} \cos \omega t dt}{x_{\infty}} \ldots [12]
\]

where

\( d \) is the length of the free path of the mass particle \( m \)

(2) the notations \( n \) and \( n + 1 \) are used to denote sequence in terms of an arbitrary cycle \( n \). In particular \( n \) could be taken equal to zero

\[
T_{n+1} = \frac{d + T_{n+1} x_0 \sin \omega t}{x_{\infty}} = \frac{d + x_0 \sin \omega T_{n+1}}{x_{\infty}}
\]

\[
d = x_0 (\omega T_{n+1} - \sin \omega T_{n+1}) = x_0 (\theta_{n+1} - \sin \theta_{n+1}) \ldots [13]
\]

Equation [13] can be used for designing efficient acceleration dampers, since it gives a simple relationship between a desired angle \( \theta \) and the depth of channel \( d \) in terms of \( x_0 \).

In calculating the periodicity of contact for a constant amplitude it is convenient to consider the following two cases separately:

Case (1): \( \theta < 180 \) deg

\[
T_c = \frac{d + \int_{n\omega}^{T_{n+1}} x dt}{x_{\infty}} + \left[ \frac{(n+1)\pi}{\omega} - T_{n+1} \right] \ldots [14]
\]

where \( T_c \) is the periodicity of contact.

But

\[
\frac{d + \int_{n\omega}^{T_{n+1}} x dt}{x_{\infty}} = T_{n+1}
\]

\[
\therefore T_c = \frac{(n+1)\pi}{\omega} \ldots [15]
\]

Since \( n \) is arbitrary, let \( n = 0 \) in Equation [15]

\[
T_c = \frac{\pi}{\omega}
\]

That is for \( \theta < 180 \) deg the particle \( m \) will contact \( M \) two times per cycle.

Case (2): \( \theta > 180 \) deg

There exists a phase angle \( \theta > 180 \) deg for which the work done per impact is equal to that done by a corresponding \( \theta > 180 \) deg. In other words, for the work done for phase angles \( \theta > 180 \) deg there exists a corresponding phase angle in the first two quadrants for which the work is the same. It follows therefore that the phase angle giving greatest damping efficiency for a single contact can be determined in the first two quadrants, i.e., \( \theta < 180 \) deg. Furthermore, since for \( \theta < 180 \) deg the periodicity of contact is 2, and for \( \theta > 180 \) deg it is less than 2, it follows from Equation [11] that the phase angle corresponding to maximum energy dissipation per cycle must be \( \gtrsim 180 \) deg.
A Criterion for Determining Phase Angle $\theta$, Giving Maximum Damping Efficiency

In the foregoing discussion it was shown that $\theta_e$ must be $\leq 180$ deg. This simplifies the problem of determining $\theta_e$ for a given amplitude of oscillation. Equation [11] can be written in the form

$$\frac{2\pi}{\omega} = -\frac{N\alpha^2x_0^2}{2} \sin^2 \theta + m\omega^2 \alpha^3 N (\cos \omega RT - \cos^3 \omega RT).$$

[16]

for $0 < \theta < 180$ deg, then $T_e = \frac{x}{\omega}$ (see Equation [15] letting $n = 0$).

Since there are two hits per cycle, $N = 2$ and Equation [16] can be written in the form

$$\frac{2\pi}{\omega} = -m\alpha^2x_0^2 \sin^2 \theta + 2m\alpha^2x_0^2 (\cos \theta - \cos^3 \theta) = -m\alpha^2x_0^2 (1 - \cos^3 \theta).$$

[17]

It is interesting to note that Equation [17] for the work done by the power supply can be arrived at directly by calculating the loss in kinetic energy of the mass particle per impact. It is

$$W = -\frac{1}{2} (x_1 - x_2)^2.$$  

[17a]

which is in agreement with Equation [18].

The reason for using the momentum approach for arriving at Equation [18] is to give a clear step-by-step picture of the mechanism of the acceleration damper and thereby facilitate handling of transient cases, effect of rebound, and extension of theory to modified cases.

The phase angles $\theta_e$ giving maximum and minimum damping efficiency can be determined by differentiating Equation [17] with respect to $\theta$ and equating the result to zero. Differentiating Equation [17] and equating to zero gives

$$\frac{dW}{\theta} = -m\alpha^2x_0^2 [2 \sin \theta \cos \theta - 4 \sin \theta \cos \theta + 2 \sin \theta] = -2m\alpha^2x_0^2 \sin \theta [1 - \cos \theta] = 0.$$  

[18]

This is satisfied by $\theta = 0$ and $\theta = \pi$, which corresponds to minimum and maximum damping efficiency, respectively. Applying this result to Equation [13] we obtain $d_e = 2\pi$, where $d_e$ is the free path of $m$ corresponding to maximum damping efficiency. Equation [17] shows that the damping efficiency increases continuously from $\theta = 0$ to $\theta = \pi$ and the work done by the particle on the system is always negative. This means that power input is required to maintain a constant amplitude of oscillation.

Calculating Motion of a Single Degree of Freedom Under Influence of an Acceleration Damper

The following development neglects the effect of friction on the motion of the mass particles in the acceleration damper. The motion can be formally represented by an integral equation of the form

$$x(t) = \int_0^t f(x(t)) dt.$$  

[19]

The solution of Equation [19] will be carried out by using step-by-step integration procedure for nonsteady harmonic motion as follows:

Fig. 4 Multunit Tube-Type Acceleration Damper

Fig. 5 Single-Unit Rectangular Tube-Type Acceleration Damper

Fig. 6 Typical Acceleration-Damper Installation in a Control Surface
Let 

\[ x(t) = x_0(t) \sin \omega t \]

\[ 1/2M \ddot{x}_1 = 1/2M (x_0(t) \omega \cos \omega t)^2 \]

\[ 1/2K x_2^2 = 1/2K (x_0(t) \sin \omega t)^2 \]

Then using Equation [17a] and substituting \( x_1 = x_0 \sin (\omega t + \theta) \) and \( x_2 = x_0 \sin \omega t \) the energy \( E(t) \) can be expressed as follows

\[ E(t) = 1/2M (x_0(t) \omega \cos \omega t)^2 + 1/2K (x_0(t) \sin \omega t)^2 \]

and

\[ x_0(t) = \sum_{i} F[x(r)] \]

where

\[ \theta \]

is determined by \( d = x_0(\theta - \sin \theta) \). (See Equation [13].)

From this the logarithmic decrement can be determined and hence the value of \( c \) in Equation [22].

Typical designs of the acceleration damper are set forth in Figs. 4, 5, and 6.

**SUMMARY OF EXPERIMENTAL RESULTS AND CORRELATION WITH THEORY**

In the experiments conducted a cantilever beam was deflected, then released and allowed to vibrate with and without damper units attached, the frequency being controlled by the length of the beam. The converging amplitudes were recorded by means of a seismograph. Figs. 11, 12, 13, and 14 are typical of the numerous recordings obtained in this manner. Fig. 15 shows test results for an acceleration damper installed on a system excited by a simple harmonic force. The effective moment of inertia of the damper used, with respect to the center of rotation, was 15 per cent of the moment of inertia of the system.

In general, the agreement between theory and experiment is good as borne out by Fig. 7, and the numerical results which are given in the next section. The efficiency of the acceleration damper is considered reduced when the friction of the moving mass is increased, as realized when using multiparticles in a single container. This can be seen by comparing Figs. 11, 12, and 13.

The theory relating to flutter has not been experimentally verified and would necessitate flutter wind-tunnel tests.

**COMPARISON BETWEEN CALCULATED AND MEASURED LOGARITHMIC DECREMENT AT AN AMPLITUDE OF 0.358 IN. AND A FREQUENCY OF 9.875 RADIANS PER SEC**

Refer to Equation [19]

\[ M = \frac{7.39 \text{ lb sec}^2}{386 \text{ in.}} = 1.056 \text{ lb sec}^2 \]

\[ \omega = 9.875 \text{ rad/sec} \]

\[ x_0(t) = 0.358 \text{ and is the maximum amplitude of the beam preceding impact. Then the energy of the beam at impact is} \]

\[ \frac{1}{2} x_0(t)^2 = \frac{1}{2} x_0(t) \times 9.875 \times 0.358 \times 0.358 = 0.12 \]

\[ \frac{1}{2} x_0(t) \times 0.358 \times 9.875 \times 0.358 \times 4 = 0.05 \]

\[ \frac{7.39}{772} (x_0(t))^2 = 0.05 \]

then \( x_0(t) = 0.23 \) where \( x_0(t) \) is the maximum amplitude of the beam following said impact.

The calculated logarithmic decrement is then

\[ \frac{0.358}{0.22} = 0.39 \]

as compared to the measured value

\[ \frac{0.358}{0.23} = 0.39 \]

In the foregoing computations the structural damping of the beam has been neglected, which is consistent with this comparison.

**CORRELATION BETWEEN ACCELERATION AND VISCOUS DAMPING**

In order to facilitate application of the acceleration damper to specific problems it is convenient to represent its effect by an effective damping coefficient \( d \) such that when multiplied by the acceleration gives a damping force in phase with the velocity. The effective damping coefficient as defined can be conveniently determined experimentally and theoretically in accordance with the following development: The energy dissipated over a given time interval by an acceleration damper can be expressed in the form

\[ -j \int_{0}^{t_f} \frac{d^2x}{dt^2} dt \]

where \( j = \sqrt{-1} \)

The problem consists of determining \( d \) and to ascertain that it adequately represents the damping characteristics of the acceleration damper. In accordance with the following energy criterion, \( d \) can be readily expressed in terms of a viscous damping coefficient \( c \) corresponding to the decay curve obtained for the acceleration damper in the time interval \( \Delta t = (t_f - t_i) \)

Proceeding in this manner gives

\[ -j \int_{t_i}^{t_f} d^2x dt = \int_{t_i}^{t_f} c x^2 dt \]

where \( c \) is the corresponding viscous damping coefficient. From Equation [21], assuming simple harmonic motion

\[ jd \int_{t_i}^{t_f} \omega^2 x_0 \sin \omega \cos \omega dt = x_0^2 \int_{t_i}^{t_f} \omega^2 \cos^2 \omega dt \]

Since

\[ jx_0 \sin \omega \cos \omega = x_0 \sin (\omega + 90) = x_0 \cos \omega \]

\[ d \int_{t_i}^{t_f} \omega^2 x_0^2 \cos^2 \omega dt = \int_{t_i}^{t_f} \omega^2 \cos^2 \omega dt \]

\[ \therefore \ddot{\omega} = \omega; \quad \ddot{d} = c/\omega \]

Equation [22] enables one to calculate \( d \) in terms of the viscous damping coefficient \( c \) corresponding to the decay curve of the acceleration damper. Expressions of the form of Equation [20] can be used to represent the damping force of an acceleration damper in the equations of motion of a given system.

**INTRODUCTION OF ACCELERATION DAMPING FORCES IN EQUATIONS OF MOTION ENCOUNTERED IN FLUTTER ANALYSIS AND EQUATIONS OF MOTION INVOLVING SIMPLE HARMONIC FORCING FUNCTIONS**

As an example, consider the coupled and uncoupled damping forces in the \( h, \alpha, \) and \( \beta \) co-ordinates corresponding to an acceleration damper installed near the trailing edge of an aileron,
where $h$, $\alpha$, and $\beta$ are the generalized co-ordinates for the un­
coupled bending torsion and control-surface modes, respec­
tively. Let $\ddot{\beta}^p$ represent the uncoupled acceleration damping
coefficient in the $\beta$ generalized co-ordinate. The coupled and

\begin{align*}
\ddot{\alpha} &= \text{uncoupled damping coefficient in } \alpha \\
\ddot{h} &= \text{uncoupled damping coefficient in } h \\
\ddot{\beta} &= \text{coupled damping coefficient between } h \text{ and } \beta \\
\ddot{\alpha} &= \text{coupled damping coefficient between } \beta \text{ and } \alpha \\
\ddot{\beta} &= \text{coupled damping coefficient between } h \text{ and } \alpha
\end{align*}

In the foregoing development it is assumed that the damper is
installed in a control surface. The damping forces corresponding
to the set of damping coefficients just given are as follows

\begin{align*}
F_{\beta\beta}^{\alpha} &= -j \ddot{\beta}^p \\
F_{\alpha\alpha}^{\alpha} &= -j \ddot{\alpha}^p \\
F_{\beta\beta}^{\beta} &= -j \ddot{\beta}^p \\
F_{\alpha\alpha}^{\beta} &= -j \ddot{\alpha}^p \\
F_{\beta\beta}^{\alpha \beta} &= -j \ddot{\beta}^p \\
F_{\alpha\alpha}^{\alpha \beta} &= -j \ddot{\alpha}^p \\
F_{\beta\beta}^{\beta \beta} &= -j \ddot{\beta}^p \\
F_{\alpha\alpha}^{\beta \beta} &= -j \ddot{\alpha}^p
\end{align*}

Equations [23] and [24] can be readily used in conventional
three-dimensional flutter analysis by spanwise integration of the
forces in accordance with an energy criterion.\footnote{Equation [23] can also be introduced as generalized forces in any system of
equations of motion. The effect of the acceleration damper on
various flutter modes of an actual airplane has been investigated
theoretically by introducing generalized damping forces of the
form given by Equation [24]. The results are promising and indi­
cate the damper to be effective, especially for high-speed flutter.}  

Conclusions

Some conclusions relating to the acceleration damper are as
follows:

1. Its application to vibration control for aircraft is promising.
2. The agreement between the theory developed herein for
the acceleration damper and available test data is good.
3. The theory provides an analytical method for designing
the acceleration damper efficiently for specific application.
4. Results obtained relating to flutter justify experimental re­
search regarding the application of the acceleration damper to
flutter control.
5. The results arrived at in this paper can be directly applied
to curvilinear motion by replacing $x_0$ by $s_0$ and $x$ by $s$.

Note: Patents on Acceleration Damper are pending. Use of this
damper requires License Agreement.

\footnote{Equation [23] by W. M. Bleakney,
Journal of the Aeronautical Sciences, vol. 9, Dec., 1941, p. 56.}
Heater Designs for the Petroleum Industry

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Present-day requirements for high-octane aviation fuels and for toluol and benzol fractions for use in the production of munitions increase the need for specialized types of oil heaters. In the past most oil heaters were designed by rule of thumb but this method gave way to more accurate empirical methods while attempts were being made to put the art of design on a sound theoretical basis. The empirical relationships used were derived from correlations of experimental and operating data, and these relationships were satisfactory when used to design heaters of types and service requirements similar to those on which the data were obtained. However, as soon as specialized requirements, such as higher temperatures, increased heat-transfer rates, different box shapes, etc., are involved, the extrapolation of empirical equations and methods becomes hazardous. For this reason strenuous efforts have been made to place all design equations and methods on a theoretical basis, and it is believed that these efforts have yielded satisfactory results. A discussion of these methods of design is the subject of this paper.

Oil heaters may be classified on the basis of the service which they are to perform, namely, as simple heaters in which no chemical conversion of the oil is to take place, and as reaction heaters, typified by cracking furnaces, in which a definite degree of conversion is desired. The basic problems are the same in the two types of heaters with the exception that in the reaction heater there is the additional requirement that the oil must be maintained for a specified time period at a given temperature, or over a given temperature range.

**Furnace Sketches**

Some of the more recent heater designs which have proved successful in meeting the particular requirements for which they were designed are shown herewith. Fig. 1 shows a simple helical-coil radiant-section furnace which was developed primarily as a low-cost low-efficiency furnace for small duties, preferably not over 10,000,000 Btu per hr. Since a tube cleaner cannot be used on this furnace, its applications should be limited to services in which no coke or scale deposits are expected. However, this type of furnace may also be used where coke is expected if the coke-burning-out procedure is used. By this method the coke deposits are removed by passing air and steam through the coil while firing it sufficiently to start and maintain combustion of the coke. Enough steam is used to keep the combustion temperature under control.

Fig. 2 shows an A-frame furnace which has been found to be a very satisfactory type for a large range of services. Developed originally as a simple heating furnace for duties ranging from 10,000,000 to 30,000,000 Btu per hr, it has since been found suitable for duties as high as 100,000,000 Btu per hr. By routing the oil flow through it properly, this type may also be used as a cracking furnace. Its greatest advantages are (a) lower cost per mil-

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1 M. W. Kellogg Company. Mem. A.S.M.E.

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.
offers about the only solution possible when a great number of equal streams of oil are to be heated equally. This type of furnace may be used for practically any service but its higher cost normally limits it to use under the conditions stated. By the use of the proper burners suitably arranged in the roof of the furnace, the heat absorption of each tube will be substantially the same as that of any other so that all the tubes may be connected in parallel.

Fig. 4 shows a convection-type furnace. This furnace has a rather limited field of usefulness. It is particularly suitable only when a large soaking volume at a rather low heat-transfer rate is required. Such a requirement is usually met in gas-pyrolysis or polymerization work. For other services where higher over-all average rates are required or desired, the furnace will not be satisfactory since the rate of heat transfer in the convection section is low owing to the low flue-gas velocity through it.

In a paper by Rickerman, Lobo, and Baker (1), several arrangements of the conventional type furnace are shown, both in the single-radiant-section type and the double-radiant-section type. This furnace had been widely used because of its flexibility in variation of size and shape. However, the overhead-convection-section type of furnace, an example of which is shown in Fig. 5 of reference (1), has essentially all the features of the conventional type but it is usually less costly. In the overhead-conve-
vection-section-type furnace the stack may be mounted on the furnace itself thereby eliminating expensive duct work and additional foundations. As a result of eliminating duct work the size of the effective portion of the stack itself may also be considerably reduced. This type of furnace may be built as a single- or multiple-radiant-section type, with or without bridge-walls and may be fired from the walls or from the floor. A triple-radiant-section type, fired from the floor, is shown in Fig. 5 herewith. Here the furnace shows a considerable reduction in first cost over the end-fired type, since, with the use of the floor for burner space, a large number of small burners may be installed with the result that considerably less clearance may safely be used between burners and tubes.

Duty or Heat Absorption

For the simple heater in which no reaction is to take place, the specified conditions such as throughput, gravity, temperatures, pressures, and phase conditions will determine the amount of heat to be absorbed by the oil in its passage through the furnace, or more briefly, the furnace “duty.” This duty will consist of the sensible heat plus the heat of vaporization, if any, and may be obtained by use of specific-heat and latent-heat curves, or more conveniently, by means of enthalpy, or total-heat curves.

In the case of the cracking heater, the heat of chemical reaction must also be taken into account and added to the sum of the sensible- and latent-heat duties mentioned. Sometimes it may be found that the heat of reaction is exothermic rather than endothermic, in which event it must of course be subtracted from the other heat duty.

Some laboratory work has been done in an effort to determine the heat of reaction for at least a few typical cracking operations, but the number of variables involved has made it very difficult to obtain a satisfactory correlation. Some approximate correlations have been made based on data obtained from furnace tests and these correlations have been used for such heaters as visbreakers, gas-oil crackers, naphtha reformers, and thermal-polymerization furnaces. Where the character or analysis of the furnace charge and products is accurately known the heat of reaction may be calculated as the difference in the heat of combustion or the heat of formation of the charge and products. However, by this method the heat of reaction is determined as a relatively small difference between two very large total quantities so the stipulation that the analyses be accurately known is an important one.

Efficiency of Furnace

Having calculated the furnace duty, the over-all requirements of the oil side of the furnace, or, as it may be considered, the high-temperature-level heat exchanger, are known. The fluid on the other side of the exchanger will consist of flue gas from the combustion of fuel. The flue gas will be available at combustion conditions and may give up its heat to the oil side until its temperature approaches that of the oil to which it is transferring its heat as it leaves the furnace. Therefore, the temperature of the oil in the last pass of the furnace and the closeness of approach of flue gas to oil temperature will practically set the efficiency of the furnace unless other heat-recovery equipment is used. Normally the oil charge will be brought into the furnace at the point where the flue gas is leaving so that with the inlet-oil temperature known and the nearest economical temperature approach fixed, the temperature of the flue gas leaving the furnace may be fixed.

The temperature approach, or difference, will obviously be set by considerations of economy and whether other heat-recovery equipment such as an air preheater, or waste-heat boiler, is to be used. When no external heat-recovery equipment is to be used it has been found that 200°F is about the nearest economical temperature approach. With low-cost fuel and low oil-inlet temperatures this difference may be as great as 600 to 800 deg F. In general, however, most oil heaters are designed for exit flue-gas temperatures between 700°F and 1100°F, with the majority between 900°F and 1000°F.

The determination of other factors will completely fix the furnace efficiency, namely, the character of the fuel to be fired in the furnace, the percentage of excess air with which it will be burned, and the radiation loss. In a natural-draft furnace it is possible to burn completely almost any oil or gas fired with 40 per cent excess air, and with care in the design and operation of the furnace even lower percentages may be obtained.

Once the efficiency of the furnace has been determined as outlined, the heat to be liberated in the furnace, i.e., the fuel to be fired, and the quantity and analysis of the flue gas produced may be calculated. Both sides of the high-temperature-level heat-exchanger are then known and the procedure is to determine the heating surface, its arrangement, and the physical dimensions of the furnace.

Determining the Tube Sizes

The first dimensions to be decided are those of the tubes. The inside diameter of the tubes will be determined by the allowable or economical pressure drop which may be taken in the coil. In some cases the effective pressure in the cracking coil may have considerable effect on the amount or nature of the thermal decomposition taking place so that a maximum pressure drop may thus be determined. In general, however, economic considerations such as pump cost and horsepower requirements will set the value of the permissible pressure drop.

Of course, until the furnace is finally designed it is not possible to calculate the pressure drop which will obtain with a given tube size, since among other things the number of tubes will not be known. However, a preliminary estimate of the number of tubes required may be made and then an approximation may be made of the pressure drop. This process may be repeated until a suitable internal diameter of tube is determined. It should also be remembered that the oil may if desired flow through several tubes in parallel.

With experience the designer usually will be able to choose an internal tube diameter without making these preliminary pressure-drop estimates. A criterion which may be used as a guide is the oil velocity through the tube, either in terms of "cold-oil velocity," that is, linear velocity of the oil in feet per second, using the density of the oil as liquid at 60°F, or in terms of mass velocity. The cold-oil velocity in most oil heaters will range from about 2 to 10 fps. For vacuum pipe stills the cold-oil velocity may be even lower, some having been designed for about 0.5 fps. For crude heaters and visbreakers the velocity normally ranges from 2 to 5 fps, for gas-oil crackers, naphtha reformers, and polymerization furnaces from 5 to 8 fps, for delayed-coking furnaces, about 7 fps, and for furnaces heating material all in the liquid phase, from 7 to 10 fps. Higher or lower velocities may also be used at an increase in the resulting pressure drop on the one hand and with a poorer inside oil-film heat-transfer coefficient on the other hand. Mass velocities may vary from 100 to 500 lb per sec per sq ft of cross-sectional tube area.

The foregoing discussion is predicated upon the assumption that the highest possible velocity "compatible with the allowable pressure drop should be used, since the higher the velocity in the tube the better the inside-film coefficient of heat transfer. This means that for a given maximum allowable tube-metal temperature or for a given maximum allowable oil-film temperature, a higher over-all heat-transfer rate may be used. The upper limit of tube-metal temperature may be set by the tube materials
to be used, or the limit of oil-film temperature may be set so as to avoid coking of the oil film or discoloration of the oil caused by overheating of the film.

Having set the approximate internal diameter of the tubes, an external diameter may be chosen to give a tube thickness appropriate for the pressures and temperatures which will obtain in the tubes.

The length of tube to be used will depend largely on the type of furnace. In general, it has been found that for a given type of furnace it is most economical to use the longest tubes commercially available while at the same time covering the walls and arch of the combustion chamber with tubes. At least two of the dimensions of the combustion chamber, or radiant section, will be determined by the number and size of burners required, so that either the number or length of tubes may be set. If the number of tubes is thus fixed by the box dimensions the length must be calculated from the required heating surface. Alternatively, if the length of the tubes is fixed, the number of them must be obtained from the surface. Actually, it is usually necessary to make trial-and-error calculations in order to obtain a well-balanced economical furnace design from the standpoint of the number and length of tubes.

Another point which should be kept in mind is that more intermediate tube supports are required for long tubes than for short, so that the tube length should, if possible, be chosen to take the greatest advantage of the intermediate tube supports which are necessary. Furthermore, it should be remembered that the pressure drop per square foot of heating surface will be less with long tubes since fewer return bends are required.

Normally the tubes chosen will have outside diameters varying from 2 to 6 in. by 1/8-in. increments between 2 and 3 in., and by 1/4-in. increments between 3 and 6 in. Occasionally smaller tube sizes, iron-pipe-size seamless tubing, or 8-in-OD tubing are used. Tube thicknesses vary from about 1/4 to 1 in., the lower limit being set by a nominal erosion rate, while the upper limit is set by the difficulty of rolling heavier walled tubes into return headers. Over-all tube lengths vary from about 20 ft to 40 ft by 2-ft increments. Shorter lengths may in some instances be used although ordinarily they are uneconomical owing to the increased number of return bends required per square foot of heating surface. Tubes longer than 40 ft may be used if obtainable, or if not obtainable in one piece, two pieces may be butt-welded to make the greater length. Maximum lengths of single-piece tubes will depend on the physical capacity of the tube mill to handle the length, or on the size of the steel billet from which the tube is drawn. A practical limit of tube length may be set by the difficulty of cleaning.

**Calculating the Radiant Section**

The next step in the design of the heater is to calculate the radiant section or sections. The heat-transfer surface required in each radiant section will depend largely on the heat-transfer rate which is desired in the section. In addition, the percentage of excess air with which the fuel is burned will have an effect. In the Wilson, Lobo, and Hottel (2) empirical radiant-heat-transfer equation for box-type radiant sections these two items are the only variables. This equation has been rather widely used but it is empirical and has a number of limitations. Chief among these is the requirement that the radiant section be of the box type as well as within certain size limits. The box should not vary greatly from a ratio of side dimensions of about 1 to 4, that is, the longest side should not be more than about 4 times as great as the shortest side. Furthermore, the cube root of the internal volume of the radiant section should be within the range of 15 to 25 ft. The average outside tube-metal temperature should be in the range of 700 to 1100 F, while the temperature of the flue gas leaving the radiant section should be in the range of about 1100 to 1600 F, but in no case should the average metal and exit-gas temperatures be closer than 400 F. In effect, these limitations set the range of radiant-heat-transfer rates between about 4000 and 12,000 Btu per hr per sq ft. The excess air, character of fuel, and distribution of heating surface should also be within the range of the data obtained in the furnace tests used in the derivation of the equation. These data are presented in the original paper (2).

In order to place the design of radiant sections on a sounder basis, a more rational radiant equation was developed by Hottel (3) and modified by Evans and Lobo (4). This equation, having a theoretical basis, should be suitable for use with any radiant section. In it account is taken of most of the variables which affect the heat-transfer rate. The equation is

\[
\frac{H}{\alpha A_{cp}} = 0.173 \left( \frac{T_s}{100} \right)^4 - \left( \frac{T_a}{100} \right)^4 + 7 (T_e - T_s) \ldots [1]
\]

where

- \( H \) = heat absorbed by radiant surface, Btu per hr
- \( \alpha = \) factor by which \( A_{cp} \) must be reduced to obtain effective cold surface
- \( A_{cp} = \) area of a cold plane replacing the tubes, sq ft
- \( \phi = \) over-all heat-exchange factor
- \( T_s = \) temperature of flue gas leaving radiant section, deg F abs
- \( T_e = \) temperature of heat-absorbing surface, deg F abs

As may be seen, Equation [1] is a modified form of the basic Stefan-Boltzmann equation, the modifications consisting of the definition of the effective heating surface \( \alpha A_{cp} \), the inclusion of a factor \( \phi \) to take account of a number of variables, and the addition of an arbitrary factor \( 7(T_e - T_s) \) to account approximately for convection heat transfer in the radiant section.

The effective heating surface has been defined by Hottel (5, 6) while the over-all exchange factor \( \phi \) is defined by Evans and Lobo (4).

In the nomenclature following Equation [1] \( T_s \) is defined as the temperature of the flue gas leaving the radiant section. This definition is true only if the radiant box and the arrangement of the burners in the box are such as to give fairly complete mixing of the flue gas so that the exit gas temperature is also practically the same as the mean gas temperature within the radiant section. For radiant sections in which the products of combustion move substantially in a straight line parallel to the heat-absorbing surface so that the flue gas has a definite temperature gradient from the initial combustion zone to the exit, it is not justifiable to use the exit temperature in the equation. In this case a mean effective radiating temperature must be determined, or the radiant section may be divided into two or more sections and the exit-gas temperature from each section used in the equation when calculating that section. Obviously, with the latter method the proper values of \( \alpha A_{cp} \), \( \phi \), and \( T_e \), corresponding to the section being calculated, should be used. Term \( T_s \) may be taken as the average outside tube-wall temperature in the section under consideration.

Another point which should be noted is that if any of the tubes in the convection section are exposed to direct radiation from the hot products of combustion in the radiant section, this exposed surface must be added to obtain the total effective radiant surface. The over-all heat-transfer rate to the exposed convection surface will then be the sum of this direct radiant rate and the convection rate to be calculated subsequently.

Obviously, the solution of a heat balance using the radiant-section heat absorption, as just calculated, and an allowance for radiation losses should give an exit-flue-gas temperature which
agrees with the assumed radiant-section exit-flue-gas temperature.

Heat-transfer rates commonly used in the radiant sections of oil-heating and cracking furnaces vary from about 5000 to 20,000 Btu per hr per sq ft of exposed outside tube surface. Very low rates may be used in a furnace without a convection section in order that a reasonable furnace efficiency will be obtained. In delayed-cooking furnaces and vacuum pipe stills, rates between 5000 and 10,000 are usual while for crude heaters, rates between 10,000 and 12,000 are used. For gas-oil cracking furnaces, naphtha reformers, and polymerization furnaces, rates of from 12,000 to 16,000 are common. Low-temperature reboilers may be designed for rates as high as 20,000 Btu per hr per sq ft. Actually even higher rates than those just outlined are often obtained in practice.

**Convective-Section Design**

With the radiant-section heat absorption calculated, the remaining duty, that to be performed in the convection section, may be obtained by difference from the total, and the oil temperatures into and out of the various divisions of the convection section may be calculated. The heat balance on the flue-gas side for the radiant sections gives the temperature of the flue gas entering the convection section, and subsequent heat balances will give the necessary intermediate flue-gas temperatures.

The length of the tubes in the convection section is usually made the same as those in the radiant section. The width of the convection bank is then set so as to obtain the best heat-transfer rate possible. On the other hand, the flue-gas velocity through the convection bank should not be too high or an excessive draft loss will result and the stack required will be uneconomically large. It has been found through considerable experience that a flue-gas mass velocity of about 0.5 lb per sec per sq ft of minimum free area usually will be satisfactory. Mass velocities approaching 1.0 will require a stack of such size that the over-all furnace cost may be greater than if a lower velocity were used with the resulting increased heating surface.

With the temperatures known on both sides of the flue-gas-to-oil "heat exchanger," the flue-gas velocity known, and most of the dimensions of the section known, the heat-transfer rates may be calculated. Heat will be transferred by both radiation and convection, in parallel, from the flue gas to the outside surface of the tube and the heat thus transferred will then pass, in series, through the tube wall, any scale or coke deposit, and the inside oil film, to the oil.

The radiant heat transfer consists of "direct" radiation from the hot flue gases and hot refractory walls of the radiant sections, for the first two or three rows of the convection section, (if they are "exposed" to the heat from the radiant section) plus radiation from the hot flue gases passing through the tube bank, and from the hot adjacent refractory walls of the convection section.

The direct radiation to the first rows is included in the radiant-section calculation as previously discussed. The radiation from the hot refractory walls adjacent to the convection tubes may be calculated by an equation for the "wall effect," as given by Monrad (7). The radiation from the hot flue gases passing through the tube bank may be calculated by the method of Haslam and Hottecl (8, 9), using the radiation charts for carbon dioxide and water vapor.

The convection-heat-transfer coefficients may be calculated by the use of Monrad's equation (7), or by the method given by Pierson, et al (10, 11, 12). It is believed that the latter method is the more accurate. In any event, it undoubtedly may be used with a greater degree of assurance in ranges beyond the range of the data upon which the former equation was based. The method which is fully described in the reference articles, consists of a correlation between the Reynolds number and the Nusselt number. In the original article curves are given relating these two numbers, and by their use the convection coefficient may readily be obtained.

**Summary of Heat-Transfer-Rate Calculations in Convection Section**

Summarizing the operations required in calculating the heat-transfer rate in the convection section, the following steps must be taken:

1. Assume or by other means fix the total heat to be absorbed in the section under consideration.
2. By heat balance, making suitable allowances for radiation losses, calculate the flue-gas and oil temperatures both into and out of the section.
3. Calculate the logarithmic-mean temperature difference between the flue gas and oil.
4. Calculate the inside-oil-film coefficient in Btu per hour per square foot per deg F. Equations for this coefficient are given by Walker, Lewis, McAdams, and Gilliland (13), by McAdams (14), and others.
5. Assume an over-all heat-transfer rate for the section.
6. Calculate the average outside tube-metal temperature by adding to the average oil temperature the temperature drop through the oil film, coke deposit (if any expected), and the metal wall of the tube. The latter temperature drop is often neglected since it normally is of small magnitude.
7. Determine the amount of radiation in Btu per hour square foot from the flue gas passing through the tube bank at its average temperature and composition, to the tube walls at their average temperature and surface emissivity by means of the radiation charts for carbon dioxide and water vapor.
8. In order to convert it to a coefficient, divide the radiant rate determined under item 7 by the logarithmic-mean temperature difference of item 3.
9. Convert to a coefficient the direct radiation, if any (from the radiant section), by dividing it also by the mean temperature difference.
10. Calculate the coefficient of convection-heat-transfer rate as outlined previously.
11. Add the three coefficients of items 8, 9, and 10 arithmetically, since the total heat flow is the sum of these three components.
12. Multiply the over-all gas-coefficient by Monrad's (7) wall-effect factor to take into account the quantity of radiation from the hot refractory adjacent to the tubes. This result will be the true over-all gas coefficient for the outside surface of the tube.
13. Calculate the coefficient of heat transfer through the metal wall of the tubes and convert this to the basis of the outside surface by multiplying by the ratio of average to outside tube diameter.
14. Calculate the coefficient of heat transfer through the expected coke deposit, if any, and convert to the outside-surface basis by multiplying by the ratio of inside to outside tube diameter.
15. Convert the inside-oil-film coefficient of heat transfer to the outside-surface basis by multiplying by the ratio of inside to outside tube diameter.
16. Add the reciprocals of the previous four coefficients. The sum will be the reciprocal of the over-all coefficient.
17. Multiply the over-all coefficient of item 16 by the mean temperature difference to obtain the average heat-transfer rate on the outside tube surface. This rate should check the assumed rate rather closely.
18 Obtain the heating surface required by dividing the assumed oil duty for the section by the calculated rate.

Items 13 and 14 of the summary are usually neglected but under certain conditions it may be advisable to include them.

Soaking Sections

In the discussion thus far only heating surface has been considered and in many of the furnaces to be designed, heating surface is all that is required. In cracking, pyrolysis, polymerization, and some other furnaces, however, some of the tubes will perform a double service. They will absorb heat as well as maintain the oil at approximately a constant temperature or within a definite temperature range for the required time to complete the reaction. These tubes will be in the last part of the coil and this part of the coil is ordinarily termed the "soaking" section or "soaker."

Since over a given temperature range, or at a constant temperature, a definite amount of heat will be required for a definite degree of conversion, it will be necessary to design the furnace so that this heat will be absorbed by the number of tubes required to hold the oil at temperature for the required time. In other words, the number of tubes in the soaking section is fixed by the soaking time required and at the same time the duty of the soaking section is fixed by the heat of reaction required, hence the heat-transfer rate in that section is fixed and the surface must be placed in the furnace so that it receives heat at that rate.

However, a considerable degree of control may in most cases be exercised by the designer over the conditions for which the soaking section is designed. For example, if the heat-transfer rate, necessitated by the desired temperature range, is too high for a consideration of the possibility of coking or for other reasons, it may be feasible to increase the working-temperature range of the soaker section while maintaining the transfer-line temperature constant. This will increase the heat duty to the section but it will increase the time or volume required for cracking to an even greater extent so that the over-all heat-transfer rate in the soaker will decrease. The number of tubes needed will obviously increase considerably. On the other hand, if the heat-transfer rate is too low the temperature rise may be decreased. Other means of control of the design are by changing the outlet temperature of the coil, providing this is possible from a standpoint of economical tube materials and the heat requirements of the fractionating system; or with limits, by changing the effective pressure in the soaking section. This change may be effected in some cases by raising or lowering the coil outlet pressure or by changing the flow through the section in order to change the pressure drop. Obviously, all the interrelated effects of any of these changes must be investigated in order to be sure that in improving one condition, a worse one does not arise at another point.

Other Considerations

Burners. In designing the radiant section or combustion chamber, space must, of course, be left available for the required number of burners in the walls or floor. This space should be large enough and the burners so located within the space that they are not too close to tube surfaces, otherwise there will be danger of direct flame impingement on those surfaces. In the case of forced-circulation water heaters, flame impingement can often be tolerated but in most oil heaters it must be strictly avoided.

Pressure Drop. In order to determine the differential pressure required of the furnace-charge pump, the thickness and material of the furnace tubes, and in the case of cracking furnaces, the amount of soaking time or volume required, it is necessary to calculate the pressure drop through the furnace coil.

The well-known Fanning equation may be used to calculate the pressure drop. This equation in a convenient form is as follows

$$\frac{\Delta P}{\Delta L} = 0.005185 \frac{f}{D} \times G^2 \times v \times \frac{1}{D} \times \frac{1}{4637P}$$

where

- $P = \text{fluid pressure, psia}$
- $f = \text{dimensionless friction factor, a function of Reynolds number}$
- $G = \text{mass velocity of fluid flowing, lb per sec per sq ft}$
- $v = \text{specific volume of fluid flowing, cu ft per lb}$
- $D = \text{inside diameter of tube, in.}$

Equation [2] makes no allowance for the pressure drop resulting from a change in the kinetic energy of the fluid. In those cases where a large fraction of the material is vaporized at low pressure (both conditions resulting in a high rate of change of specific volume and therefore of velocity), this pressure loss may be of importance. In order to include this loss the pressure-drop equation has been modified to the following form

$$\frac{\Delta P}{\Delta L} = \frac{0.005185 f G^2 v}{D} \left[ \frac{1}{1 - \frac{G^2 v}{4637 P}} \right]$$

where

- $P = \text{fluid pressure, psia}$

Other nomenclature as in Equation [2]

The calculation of the pressure drop through the coil consists of a series of trial-and-error steps. If the coil-outlet pressure is fixed, which is usually the case, the pressure drop per foot at the outlet may be calculated directly. With this figure as a guide an estimate may be made of the pressure drop for a given equivalent length of tubing, working toward the inlet of the coil and thus an estimated pressure may be obtained for this new point, $\Delta L$ feet from the outlet of the coil. Using the estimated pressure, a new $\Delta P/\Delta L$ should be calculated for this point. The logarithmic mean of the first and second $\Delta P/\Delta L$ values may then be multiplied by the equivalent length between the two points to obtain the calculated pressure drop between the points. This calculated value should check the estimated value rather closely. In this stepwise fashion the pressure drop may be calculated through the coil.

Tube Calculations. Once the temperature and pressure gradients through the coil are known the tube thickness and the tube material may be calculated. Bailey's (15) creep-stress method for calculating tubes is probably the most commonly accepted method at present. In this method the maximum shear stresses in the tube are calculated and compared with allowable shear stresses which will produce a given maximum rate of creep of the tube material.

Flue-Gas Ducts and Stack. After the heater itself has been designed as outlined the flue-gas disposal system must be considered. The simplest is a natural-draft system consisting merely of flue-gas ducts and a stack. Natural draft is used to a much greater extent than forced or induced draft in the petroleum industry. A discussion of the several systems is given by Rickerman, Lobo, and Baker (1).

The purpose of the stack and duct work is two fold; (a) to draw the required amount of combustion air through the burners, and (b) to dispose of the flue gases into the atmosphere so that they are not objectionable. The draft required at the burner throat to insure drawing the proper amount of combustion air through...
the burner will depend on the design of the burner itself. However, for those burners designed for natural-draft operation a draft of from 0.10 to 0.20 in. of water will usually be sufficient. Normally, the draft needed at the burners will be used as a starting point in calculating the total draft or stack height required. Another important point which should not be overlooked is that a slight draft should exist throughout the furnace setting for most satisfactory operation.

BIBLIOGRAPHY

Discussion

K. W. Fleschler. It is felt that Mr. Rickerman's paper is a timely and outstanding contribution to all who wish to design tube stills from a more theoretical standpoint. The author should be commended, not only for consolidating so much practical formulation and experience into one treatise, but also for a very sound and workmanlike pattern of approach to a notoriously complicated class of furnace-design problems. Mr. Rickerman's work will no doubt benefit many in the oil-refining industry; and, conceivably, could materially help equipment designers throughout the fast-growing field of "liquid heat-treatment," as involved in rayon, synthetic rubber, food, and all manner of chemical process industries.

However, in connection with techniques of firing radiant sections of tube stills, the author does not refer to one new point of view which promises radical influence on tube-still design.

Mr. Rickerman observes that "burners should not be too close to tube surfaces, otherwise there will be danger of direct flame impingement on those surfaces." He also describes long-standing problems the designer has faced in providing for: large combustion chambers; special tube-burner space relationships; and an average requirement of "40 per cent excess air to burn completely most oils or gases"—all in order to avoid nonuniformities of heating and hot spots on the coils.

Ceramic-cup gas burners are now appearing on the scene which (a) contain the combustion reaction wholly within the cup concavity, (b) may be closely faced (within 10 to 24 in.) of tubes or work without flame impingement or hot spots, and (c) may be distributed in any number and in any desired pattern over any furnace wall or roof surface—so that the heat transfer to any portion (however small or large) of a tube-still coil may be independently manipulated to establish any desired shape of the time-temperature heating curve of the oil. In a 50,000,000-Btu heater as many as 200 burners of this type can be distributed over two walls 40 X 20 ft each—each wall only 1 ft 9 in. from the coil it faces.

It is also possible through this firing method to utilize completely premixed gas-air fuel supplies at any burner pressure from a few ounces to 2 or 3 psi gage. With the normal inspirator-type burner, using gas at the orifice (or spud) at a pressure in the neighborhood of 10 to 15 psi gage as well as an even considerable stack draft, actual pressure drop across the combustion ports could not exceed a few ounces at the most. Thus firing with premixed fuel, at burner pressures measured in pounds, gives greater turndown range, i.e., more individual-burner heat output adjustability than heretofore feasible. Also, it seems that with the multiple-ceramic-concavity firing method, it is no longer necessary to provide for more than 5 or 10 per cent excess air if indeed, any at all ultimately proves to be necessary. Thus the calculations described by Mr. Rickerman will yield different
heat-transfer results than have been average industry experience to date. Also stacks will become unnecessary.

It is also important to realize that in thermal cracking or other operations in which long soaking times (at given temperatures with carefully balanced small heat inputs) are desired, it is only necessary to throttle the cup burners directed at this coil section—possibly eliminating all necessity for separating soaking chambers from other sections.

Fig. 1 illustrates the considerable simplifications and size reductions of tube-still structures possible with the type of firing described.

Author’s Closure

In general, Mr. Fleischer's comments on the advantages of short-flame burners are valid, although at the same time it must be pointed out that the methods of calculating or designing the furnace do not change at all when such burners are used. The factors or values used in the design equations do change of course, since the equations do take into account all or most of the variables affecting the design.

As to clearances required to avoid flame impingement, obviously they will depend on the expected flame length of the burners used, and experience with the burners under consideration is the best guide to the designer. Nevertheless, it is quite true that by using certain types of burners, such as the ceramic-cup type, or by using a great number of small burners, the flame length is decreased and clearances may accordingly be reduced. The designer must bear in mind the change in over-all furnace cost which may result, on the one hand with a large combustion chamber and a low-cost burner installation, and on the other hand with a smaller combustion chamber and a high-cost burner installation. He must also keep in mind the auxiliary equipment (such as air blowers) required for some of the special types of burners, and the operating and maintenance cost of both the burners and auxiliary equipment.

Better control of the heating gradient can undoubtedly be obtained by using a large number of small short-flame burners in a small combustion chamber; however, that better control may also entail much closer operational attention in order that the heating gradient may be maintained as desired.

"Turndown ratio" for burners used in the petroleum industry is usually of lesser importance than for burners used in some other industries, except as it enables control of the heating gradient.

The excess air required to burn the fuel is of importance in that it affects the "radiant-section efficiency" or ratio of radiant-section duty to heat liberated and of greater importance, the over-all furnace efficiency. The ceramic-cup burner undoubtedly will operate satisfactorily at lower percentages of excess air than the types of burners commonly used in the petroleum industry.

Mr. Fleischer suggests the elimination of the stack with the furnace which he illustrates. Normally the stack may not be dispensed with because of the second requirement of a stack—that it "dispose of the flue gases into the atmosphere so that they will not be objectionable."

Mr. Fleischer's furnace using the ceramic-cup burners is very interesting and it is felt that this type burner will be used more widely in the future in furnaces of the type shown, or in other arrangements which take advantage of the desirable features of these burners.
Cavitation in Centrifugal Pumps

By A. J. Stepanoff,1 Phillipsburg, N. J.

With the introduction of high-head centrifugal pumps of high specific speed, the problem of cavitation became of the utmost importance. As a result, extensive study and experimental work have been done in this field, mostly in connection with hydraulic turbines and with devices without moving parts. In this paper the present state of information on cavitation is presented as it applies to centrifugal pumps, with a method for determining cavitation conditions from velocity considerations. The discussion is illustrated by curves and diagrams. The model test laws as applied to cavitation are deduced, together with their limitations. Theoretical relationships governing cavitation conditions permit establishing means to avoid cavitation, which have been confirmed by experience. A summary of all important conclusions from the recent extensive literature on the subject is presented, and in conclusion, an original explanation is offered of the nature of local high destructive pressures during cavitation and in cases of metal failure by fatigue in the presence of liquids.

Introduction

In the last decade no other phase of hydraulic-machinery design and operation has been given so much attention in the technical literature as cavitation. The reason for this has been the use of higher specific speeds, both for hydraulic turbines and centrifugal pumps, with the increased danger of cavitation. To cope with the problem, experimental and theoretical studies of cavitation have been made on hydraulic turbines, centrifugal pumps, and apparatus without moving parts such as Venturi-shaped water conduits. As a result of the study and accumulated experience, modern pumps operating at higher speeds are now safer against cavitation damage than in the recent past. In this paper, an attempt is made to present a summary of the acquired knowledge on cavitation as it applies to centrifugal pumps.

Definition of Cavitation

The term “cavitation” refers to conditions within the pump, where, due to a local pressure drop, water-vapor-filled “cavities” are formed, which collapse as soon as such vapor bubbles reach regions of higher pressure on their way through the pump. In order to form such vapor cavities, the pressure first has to drop to the vapor pressure corresponding to the prevailing water temperature. The liberation of air or the formation of air- or gas-filled cavities, however, is not sufficient to produce cavitation, as the effect of air bubbles on the performance and behavior of the pump is different.

Cavitation should be distinguished from “separation,” which is a separation of the streamlines from the low-pressure side of the vane and the formation of a turbulent wake behind the vane. Separation is possible only with real viscous fluids, while cavitation is possible with hypothetic perfect liquids too. Experimentally, separation has been found to be able to exist without cavitation, and cavitation without separation. Although centrifugal fans work on the same principle as centrifugal pumps, the former can have separation while the latter can have both separation and cavitation. Cavitation can appear along stationary parts of a hydraulic machine or along a moving vane, as in the case of centrifugal-pump impellers.

The reduction of the absolute pressure to that of vapor tension may be either general for the whole system or just local which may be realized without a change of the average pressure. The general pressure drop may be produced by (a) an increase in the static lift of the centrifugal pump; (b) a decrease of the atmospheric pressure with a rise in the altitude; (c) a decrease of the absolute pressure on the system, as in the case of pumping from vessels under vacuum; or (d) an increase of the temperature of the pumping liquid, which has the same effect as the decrease of absolute pressure of the system.

The local decrease of pressure is caused by one of the following dynamic means: (a) An increase of velocity by speeding up the pump; (b) a result of separation and contraction (viscosity); and (c) a deviation of streamlines from their normal trajectory, such as takes place in a turn.

Cavitation may also be caused by a sudden starting, and stopping, and recoil of the water column, such as occur during water hammer phenomena. This type of cavitation is of a transient character and is of slight importance in centrifugal-pump practice.

Signs of Cavitation

Cavitation is manifested by one or several of the following signs, all of which adversely affect the pump performance and may damage pump parts in severe cases:

(a) Noise and Vibration. This is caused by the sudden collapse of vapor bubbles as soon as they reach the high-pressure zones within the pump; the bigger the pump, the bigger the noise and vibration. While these signs of cavitation may appear in the normal operating range of the pump only if the suction head is not sufficient to suppress cavitation, noise and accompanying vibration are present in all pumps to a varying degree when operated at points far removed from the best-efficiency point due to a bad angle of attack at entrance to the impeller. By admitting small amounts of air into the pump suction, noise can be almost completely eliminated. In this way the air serves as a cushion when the vapor bubbles collapse. This method, however, is not often resorted to for elimination of noises in centrifugal pumps, although it is an established procedure with water turbines and large butterfly valves where air is admitted automatically at partial loads (1, 2, 29).2 The beneficial effect of air admission to the pump suction under cavitation conditions is not limited to the elimination of noise and mechanical vibration, for the impeller-vane pitting is also reduced if not entirely eliminated, as this is caused by the mechanical shock accompanying the collapse of the vapor bubbles.

(b) Drop in Head-Capacity and Efficiency Curves. This appears to a different degree with pumps of different specific speed. With low-specific-speed pumps (up to 1500), the head-capacity, the efficiency, and the bhp curves drop off suddenly.
when cavitation is reached, Figs. 1 and 2. With higher-specific-speed pumps (1500-5000), however, the head-capacity and the efficiency curves begin to drop along the whole range gradually before the point of sudden breakoff is reached, Fig. 3. The degree of drop in the head-capacity and efficiency curves depends on the specific speed and on the suction pressure, increasing for higher specific speed and lower suction pressure.

With high-specific-speed pumps (above 6000) of the propeller type, there is no definite breakoff point on the curves, Fig. 5; instead, there is a gradual drop in the head-capacity and the efficiency curves along the whole range. In those types of pumps, the drop in the efficiency appears before there is a perceptible drop in the head-capacity curve. Therefore a drop in the efficiency is a more reliable criterion of approaching cavitation conditions. Even the objectionable noise may not appear until cavitation has progressed beyond the point where the efficiency becomes unsuited commercially.

Variations in the behavior of pumps of different specific speeds result from differences in impeller design. Low-specific-speed impeller vanes form a definite channel, the length of which depends on the vane angles, the number of vanes, and the ratio of the impeller-eye diameter \( D_1 \) to the impeller outside diameter \( D_2 \), Fig. 7. When the pressure at the impeller eye reaches the vapor pressure, usually on the back side of the vane entrance tips, it extends rapidly across the entire width of the channel \( A-B \), Fig. 7(a), with a small increase in capacity and decrease in head. A further drop in the discharge pressure does not produce any more flow, as the pressure differential moving water to the impeller eye cannot be increased any more. This differential is fixed by the suction pressure outside of the pump, and the vapor pressure across the whole channel between any two vanes at the impeller entrance.

With high-specific-speed impellers, the channel between two vanes is wider and shorter, Fig. 7(b). It requires more drop in head and increase in capacity to extend the vapor-pressure zone.
across the entire channel. Therefore the drop in the head-capacity curve extends through a wider range before the sudden breakoff occurs. With propeller pumps, the vanes do not overlap, Fig. 7(c). Therefore although the low-pressure zone extends when the pump head is reduced, there are always parts of the channel which remain at pressures higher than the vapor pressure, and the flow through the impeller will steadily increase even though cavitation has definitely set in.

With low- and medium-specific-speed pumps, a reduction in capacity, instead of an increase, is frequently observed at reduced discharge pressure under cavitational conditions, Fig. 13. This is caused by a further increase of the low-pressure zone along the impeller channel, and the expansion of air in the vacuous pockets. In multistage pumps, cavitation affects only the first stage; therefore the drop in head capacity and efficiency is less pronounced than in a single-stage pump. The cutoff capacity is determined by the first stage.

The drop in the head-capacity and the efficiency curves may begin before the vapor pressure is reached in certain parts of the impeller suction. This is caused by the liberation of air or light fractions in petroleum oils at reduced pressures in the impeller eye. The absolute pressure in the vacuous pockets is the sum of all the partial pressures of the gases occupying this space, in accordance with Dalton's law of partial pressures.

The drop in the head-capacity and the efficiency curves, due to liberation of free air in the water, is followed by a reduction in the bhp also. This method has been suggested (3) as a means to reduce the head and at the same time save power instead of throttling the pump discharge, as is done ordinarily. Fig. 10 is a reproduction of test results by Siebrecht, showing the head-capacity, efficiency, and bhp curves of a pump with different volumes of air admitted to the pump. The author does not know of any case where this method was employed in actual installations, but a modification of this method whereby the pump suction is throttled instead of the discharge to reduce the head, is frequently employed.

At reduced suction pressures, air or gases begin to be liberated from the liquid, producing a lower head-capacity curve and lower bhp. However, this method is not recommended because if suction-throttling is carried too far, cavitation will start...
with all its bad effects, i.e., noise, pitting, and vibration. Fig. 11 shows a test of a 5-in pump with the suction and the discharge throttled. A comparison of the bhp curves, Fig. 11, shows the power saved by suction-throttling (4). On several occasions, it has been found by careful tests on centrifugal pumps and water turbines, that the efficiency may show a slight increase shortly before cavitation sets in, Fig. 13. This is explained by a reduction of friction at the beginning of separation, just before the disturbing water-hammering begins (5, 9, 12, 22).

(e) Impeller-Vane Pitting and Corrosion-Patigue Failure of Metals. If a pump is operated under cavitating conditions for a sufficient length of time, impeller-vane pitting appears, the amount of metal lost depending on the material in the impeller and the degree of cavitation. Foettinger (5) showed conclusively that vane pitting is caused solely by the mechanical (water-hammer) action of collapsing vapor bubbles, and that electrolytic and chemical action is entirely insignificant in this process. He proved this by producing cavitation in a Venturi made of neutral glass which was pitted in the same manner as the metal in a centrifugal-pump or water-turbine impeller vanes. If electrolytic or chemical reaction is active, it should affect all the parts of the same material and not only the spots subject to cavitational water hammer.

The fact that air or gases may be more active at the instant of liberation has been stated in the past. However, the places affected by pitting are always beyond the low-pressure points...
where the vapor bubbles are formed. Another corroboration of the mechanical nature of the metal destruction has been shown by the damage of a lead plate without any loss of weight (6).

By experience it has been found that, when the collapse of vapor bubbles takes place entirely surrounded by the stream of liquid, it is harmless (6). In addition to metal destruction due to the fatigue of the metal surface as a result of repeated waterhammer blows, Poulter (7) has shown that metal particles can be torn off and carried away by liquid penetrating into and escaping from the pores of the metal under successive pressure waves. In that case, more porous materials are more readily affected by such destruction. The degree of destruction depends on the time the specimen is under pressure or the time between two successive pressure waves.

There seems to be no correlation between the hardness and cavitation erosion of metals, but apparently the molecular size and the viscosity of liquids play an important part in cavitation pitting.

Cavitation pitting should be distinguished from "corrosion" and "erosion." The first is caused exclusively by chemical and electrolytic action of the pumped liquids; the second is the wearing away of the metal parts in a pump by foreign bodies carried by the pumped liquids, such as sand, grit, coke, and coal. There is no difficulty in distinguishing between these three kinds of pitting by the appearance of the attacked parts and their location in the water passages of the pump.

Frequencies of hammering were recorded from 600 to 1000 cycles per sec by Hunsaker and up to 2500 cycles per sec by de Haller (8). The intensity of hammering depends on the velocity. Pressures of 300 atm were measured by de Haller. Local pressures confined to very small areas (1.5 mm was the piston area of de Haller's pressure-measuring device) may be considerably higher than those recorded. A satisfactory explanation of how such high pressures may arise in the case of cavitation has been lacking.

In the light of Poulter's investigation (7), it may appear possible that high destructive pressures are derived from the elastic forces of metal parts extending over areas larger than those actually attacked by cavitation. These parts are under fluctuating forces of great magnitude so great that often the whole foundation supporting the pump is set in vibration under cavitation conditions. Under fluctuating stresses, liquid is drawn in and squeezed from the pores, and it is during this squeezing phase that tremendous pressures may be produced in small restricted areas.

Similar mechanism can be applied as a partial explanation of what is known as "corrosion fatigue" of metals, or metal failure under repeated stresses in the presence of liquids. The "corrosive" effect of water, as compared with oils in the case of "corrosion fatigue," is due to the fact that water molecules are smaller than those of oil; therefore water penetration of metals would be deeper than that of oil; hence the destructive effect on the metal is greater where it is subjected to rapidly fluctuating stresses. This will explain the failure by "corrosion fatigue" of noncorrosive high-chromium steels in the presence of water. Another illustration and proof that the penetration of metals by liquid plays an important part in metal destruction by "corrosion fatigue" is furnished by results of laboratory tests by McKay and Worthington (30). They have found that the endurance limit depends not only on the stress level and the total number of cycles, but also on the frequency of stress reversals. For the same total number of cycles, the low frequency gives much lower endurance limit because more time is allowed for liquid penetration of metal with, consequently, higher destructive pressures developed in the metal pores when the liquid is compressed on the stress reversal.

### Materials to Resist Cavitation Pitting

Different materials resist cavitation pitting to a different degree. In addition to the chemical composition, the heat-treatment of metals and also the surface conditions control the amount of material destroyed by cavitation. The behavior of metals under cavitation parallels that under "corrosion-fatigue" conditions. Any notches, nicks, scratches, flaws, or sharp corners on the surface of metals attacked by cavitation accelerate the beginning of pitting. Protective coats do not improve the resistance of metals to cavitation pitting.

H. Schroeter (10) has run tests on different materials under cavitation in a Venturi-shaped conduit built for the purpose. Table 1 gives results of his tests. A velocity of 197 fps was maintained throughout these tests. The accelerated rate of metal destruction can be seen by comparing the amount of metal lost after 15 hr and 44 hr of the same materials. Fig. 15 also shows some materials tested by Schroeter.

Hardening decreases the rate of metal destruction, although

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<table>
<thead>
<tr>
<th>Metal</th>
<th>Approximate composition</th>
<th>Volume loss, cu mm</th>
<th>Brinell hardness</th>
<th>Strength, psi</th>
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<tr>
<td>Krupp steel, after 44 hr:</td>
<td></td>
<td></td>
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<tr>
<td>1 Cast steel</td>
<td></td>
<td>66</td>
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<td>2 Carbon steel, hardened</td>
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<td>3 Al 6357 annealed</td>
<td></td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Special nitrided steel</td>
<td></td>
<td>22</td>
<td></td>
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</tr>
<tr>
<td>5 Al 6357 hardened</td>
<td></td>
<td>25</td>
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<td></td>
</tr>
<tr>
<td>6 Nitro steel</td>
<td>Ni, low; Cr; Ni, 15</td>
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<tr>
<td>7 Al 6357 case-hardened</td>
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<tr>
<td>8 V2A forged</td>
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<tr>
<td>9 V2A forged</td>
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<tr>
<td>10 Cr 1348 hardened</td>
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**Bronzes, after 18 hr:**

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<th>Brinell hardness</th>
<th>Strength, psi</th>
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<tr>
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<td>Cu, 86; Pb, 10; Sn, 4</td>
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<tr>
<td>3 Alumbrine bronze</td>
<td></td>
<td>34</td>
<td>148</td>
<td>80000</td>
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<tr>
<td>4 Correx bronze</td>
<td>Cu, 88; Al, 9; Fe, 3</td>
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<td>106</td>
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</tr>
<tr>
<td>5 DB 10 (Krupp)</td>
<td>Cu, 84</td>
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<td>55</td>
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**Bronzes, after 18 hr:**

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<th>Approximate composition</th>
<th>Volume loss, cu mm</th>
<th>Brinell hardness</th>
<th>Strength, psi</th>
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<td>4 Alumbrine bronze</td>
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<td>83</td>
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<tr>
<td>5 Cr 1348 hardened</td>
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<tr>
<td>6 DB 10 (Krupp)</td>
<td>Cu, 84</td>
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</tr>
<tr>
<td>7 DB 10 (Krupp)</td>
<td>Cu, 84</td>
<td>1.1</td>
<td>106</td>
<td></td>
</tr>
<tr>
<td>8 Cr 1348 hardened</td>
<td>Cu, 84</td>
<td>0.5</td>
<td>95</td>
<td></td>
</tr>
</tbody>
</table>
Kerr (31) has tested 80 materials for cavitation in sea water in a special vibratory apparatus developed by the Massachusetts Institute of Technology. These tests show that cavitation damage with sea water was slightly greater than with fresh water. It has been found also that temperature of water has a marked effect on the metal loss by cavitation, the loss increasing with temperature. At higher temperatures the amount of air dissolved in water is reduced, thus reducing the cushioning effect of water-hammer blows, at the same time the increased vapor pressure tends to increase the vapor bubble formation.

Mousson (32) has found that loss of metal by cavitation is approximately proportional to vapor pressure. He also demonstrated the beneficial effect of admission of small amounts of air on the metal damage by cavitation. Mousson and Kerr give extensive test data which is useful in selection of materials when cavitation is expected.

**Examples of Metal Attack by Cavitation**

In pumps of normal design, the lowest pressure occurs on the back side of the impeller vane slightly beyond the suction edge. The cavitation pitting appears somewhat farther upstream where the vapor bubbles collapse, Figs. 8(a), 9(a), 9(b). However, if the pump is operating continuously at a capacity considerably higher than normal, the pitting may appear on the front side of the vane at the suction-vane tips, Fig. 6(a). Cavitation in this case accompanies separation resulting from a bad angle of attack.

Fig. 8(b) shows vane and shroud pitting near the outer shroud, due to lack of streamlining.

Fig. 8(a) shows vane pitting at the impeller discharge caused by the vane’s blunt discharge tips.

Fig. 8(c) shows a “marginal” cavitation observed on propeller pumps and also on centrifugal pumps with open impellers. Local high velocity through the clearance, and separation due to a sudden change in direction produce the “marginal” cavitation and pitting. Rounding off of the high-pressure side corners of vanes eliminates the marginal pitting at the expense of increased leakage through the clearance (12).

Fig. 9(a) shows pitting of the volute casing of a propeller pump caused by lack of streamlining.

Fig. 9(b) shows a diffusion-casing vane pitting due to a discrepancy between angles of incoming flow and diffusion vane.

Fig. 9(c) shows pitting of the tongue of a volute casing observed when a pump is operated continuously at capacities above the normal.

Fig. 9(d) is an example of pitting of the baffle in the suction nozzle, permitting excessive prerotation of the flow before it enters the impeller eye.

In general, sudden change in direction, a sudden increase in area, and lack of streamlining are responsible for local pitting of pump parts. This may appear only if the suction pressure is reduced below a certain minimum. On the other hand, pitting of pump parts on the discharge side of the pump has been observed when the pump pressure is not high enough to suppress cavitation.

**Theoretical Relationship at Cavitation Conditions**

The flow to the impeller of a centrifugal pump is produced by the existing pressure difference between the suction pressure and the pressure established by the flow at the impeller eye. The latter is not uniform at any section of the impeller passages, and even determination of the average pressure inside the impeller presents difficulties. For that reason, the theoretical relationship for the flow through the impeller eye, while easy to establish, does not give a reliable tool for an accurate predetermination of cavitation conditions. An examination of the theoretical
formulas for the flow through the impeller eye, however, enables one to learn the effect of several factors upon cavitation. A study of theoretical relationship has also resulted in the introduction of simplified formulas incorporating experimental coefficients which permit prediction of a pump's behavior as to cavitation if experimental data are available on similar pumps.

Let $H_a$ be the absolute pressure prevailing at the surface of the pump-suction supply. This will be atmospheric pressure if the suction vessel is open to the atmosphere. If the suction is taken from an enclosed vessel, $H_a$ is the absolute pressure in this vessel.

$h_s$ = static head in suction vessel above pump center line.

If it is suction lift, it is negative.

$h_v$ = vapor pressure at prevailing water temperature

$h_L$ = head loss in the suction pipe and impeller approach

$c_i$ = average absolute velocity through impeller eye (Fig. 6)

$\frac{v_1^2}{2g}$ = local pressure drop below average at point of cavitation.

Here $v_1$ is average relative velocity at entrance, and $\lambda$ is an experimental coefficient.

This local pressure drop is caused by the difference in pressure on the leading and trailing sides of the vane. When pressure is applied by the vane on water, the water exerts an equal reaction in the opposite direction, which exists as a pressure difference on the two faces of vanes. This is frequently referred to as a "dynamic depression."

Fig. 19 shows a typical pressure distribution inside an impeller channel obtained by Uchimaru (28) under actual operating conditions.

Evidently cavitation starts when

$$H_a + h_s = h_L + h_v + \frac{c_i^2}{2g} + \frac{v_1^2}{2g} + \lambda \frac{v_1^2}{2g} \quad [1]$$

When liquid in the suction vessel is boiling, the pressure in the vessel $H_a$ is equal to the vapor pressure or $H_a = h_v$ and Equation [1] becomes

$$h_s = h_L + \frac{c_i^2}{2g} + \frac{v_1^2}{2g} \quad [2]$$
meaning that a positive suction head \( h_s \) is necessary to produce the flow. To prevent vaporization, an excess of suction head is necessary above \( h_s \).

Equation [1] is not suitable for determining the maximum permissible suction lift for a given pump capacity \( (c_i \text{ and } w_i) \), because the true value of maximum \( c_i \) is not known, and also because the value of \( \lambda \) varies for pumps of different specific speed. Even for a given pump at constant speed, \( \lambda \) varies with capacity, being a minimum near the best-efficiency point and increasing on both sides of this point. Fig. 17 shows a typical curve of \( \lambda \) variation obtained by Fritz Krisam (13). Similar curves were published by von Widder (14). The increase of \( \lambda \) on both sides of the best-efficiency point shows the effect of the "angle of attack" Fig. 6(a), between the direction of relative velocity and the vane angle at the impeller entrance.

**Factors Affecting Cavitation**

A study of Equation [1] permits making a number of conclusions which hold in practice, for at cavitation conditions, a change in one term of the equation is always followed by a change in another to satisfy the relationship, thus:

(a) If atmospheric pressure is decreased due to an increased elevation (about 1 ft per 1000 ft of elevation), the pump maximum capacity will decrease \( (c_i \text{ and } w_i \text{ will decrease}) \).

(b) If suction lift is increased \( (h \text{ greater}) \), or vapor pressure rises due to higher temperature of water, pump maximum capacity will decrease.

(c) Higher suction lifts may be possible with low velocities \( (c_i \text{ and } w_i) \) or with a minimum loss \( h_s \) in the suction pipe.

(d) Note that \( H_2 \) expressed in feet of liquid depends on the specific gravity of the liquid. Thus when pumping molten salt, (used as a heating medium in the petroleum-refinery process), of specific gravity of 1.75, and the suction vessel under atmospheric pressure, \( H_s = 19.4 \text{ ft} \). Therefore, the danger of cavitation is much greater with heavier liquids (15). Vapor pressure should not be overlooked with liquids different than water.

(e) For given average velocities, \( c_i \text{ and } w_i \), the approach of cavitation is affected by the casing and impeller design as they affect the velocity distribution. Thus pump-suction design permitting more prerotation in the impeller eye will lower the maximum capacity for a fixed suction pressure. Any lack of streamlining in the suction passages of the pump and impellers results in the formation of dead water pockets (separation), increasing local velocities beyond the average or those obtained from the velocity triangle.

(f) With pumps of high specific speed of the straight-propeller type, the beginning of cavitation is indicated by a gradual drop of pump efficiency without any sudden drop in head capacity. In this case a further reduction in the suction pressure extends to the region affected by cavitation, and Equation [1] does not apply, as the vapor pressure is reached locally only; the impeller vanes do not form an entirely enclosed channel, and Bernoulli's equation (Equation [1]) cannot be used.

(g) The presence of gases in the liquid does not affect the validity of Equation [1], except that, according to Dalton's law of partial pressures, the vapor will behave as if it occupied the voids alone, and vaporization will begin at absolute pressure higher than its normal boiling point corresponding to the existing temperature. Petroleum oils represent the most complicated example of that. Being a mixture of different individual hydrocarbons, each having its own vapor pressure, light fractions will vaporize at pressures far above their normal boiling points, but the vaporization will affect only a small portion of the total flowing volume. As a result, the drop in the \( Q-H \) curve is more gradual with oils than with water, and the mechanical disturbance is not so violent. The fact that vaporization and condensation during cavitation require a heat exchange, tends to slow down the bubble formation in oils, as compared with water on account of the lower heat conductivity of oil.

(b) When studying cavitation, the suction pressure should be specified or measured at the pump-suction nozzle. In this way the loss in the suction pipe and the entrance loss are eliminated. The loss in the suction nozzle is negligibly small due to a low velocity, short distance, and accelerated flow in a normal nozzle design.

(i) In small pumps of low specific speed, the term \( \frac{c_i^2}{2g} \) is predominant in setting up cavitation conditions, and the term \( \frac{w_i^2}{2g} \) is of little significance. In high-specific-speed pumps, approaching propeller-pump type, the term \( \frac{w_i^2}{2g} \) is the controlling factor, \( c_i^2 \) is of secondary importance. Term \( \frac{w_i^2}{2g} \) depends on the pump head (and hence speed) and number of impeller vanes, decreasing with smaller head or speed and greater number of vanes.

With low-specific-speed pumps, the maximum capacity for a given suction head can be increased by cutting away part of the vanes in the impeller eye and filing the vane tips, thus increasing the available area for \( c_i \). With propeller pumps, increasing the number of vanes will improve the cavitation conditions of the pump, Fig. 18, for a given submergence, permitting higher head without noise or drop in efficiency.

(j) With low-specific-speed pumps, the maximum absolute velocity \( c_i \) may be reached either at the impeller eye (section \( A-B \), Fig. 12), or at the vane entrance. The actual effective area of both sections is greatly affected by the impeller-approach design. When studying the cavitation test data, both sections should be investigated.

**Predetermination of Cavitation Conditions From Velocity Considerations**

From a great number of observations on pumps of low and medium specific speeds (up to 1500), the author has found that for cold water, 70°F, Equation [1] can be simplified to

\[ 30 - h_s = 2.4 \left( \frac{c_i^2}{2g} \right) \]

Where \( c_i \) is the meridional velocity through the impeller eye at cutoff capacity (plane \( A-B \), Fig. 12) at suction lift \( h_s \) taken at the suction nozzle and referred to the pump-shaft center line. The same relationship is represented by a curve in Fig. 12.

A similar curve was plotted for meridional velocities at the vane entrance tips and is shown in Fig. 12. This curve can be expressed by an equation

\[ 30 - h_s = 3.5 \left( \frac{c_i^2}{2g} \right) \]

Eq. 12 also shows a curve given by Defeld in his book on centrifugal pumps (16). Velocities shown by this curve have been greatly exceeded in modern pumps.

Comparing Equations [2] and [3] with Equation [1], the following remarks can be made:

(a) The difference between the atmospheric pressure \( H_2 = 34 \) and 30 in Equations [2] and [3], or 4 ft, includes 0.85 ft vapor pressure at 70°F water temperature; the loss of head \( h_s \) in the suction nozzle; local drop in pressure due to uneven velocity distribution in the impeller approach, and a small margin of safety.
The right-hand term in Equation [2] can be expanded as follows

\[
\frac{2.4 \left( c_i \right)^2}{2g} = \frac{c_i^2}{2g} + \lambda \frac{w_i^2}{2g} = \frac{c_i^2}{2g} + \frac{\lambda}{2g} \left[ \frac{1}{(\sin \alpha)^2} + \frac{\lambda}{(\sin \beta)^2} \right] \ \ [4]
\]

Where \( \alpha \) is the absolute velocity angle and \( \beta \), the vane angle at entrance, Fig. 6. Similarly in Equation [3], the right-hand terms can be represented as follows

\[
3.5 \frac{(c_i)^2}{2g} = \frac{c_i^2}{2g} + \frac{\lambda}{2g} \frac{w_i^2}{2g} = \frac{(c_i)^2}{2g} + \frac{\lambda}{2g} \left[ \frac{1}{(\sin \alpha)^2} + \frac{\lambda}{(\sin \beta)^2} \right] \ \ [5]
\]

Term \( \delta \) is a contraction coefficient to account for the vane thickness, as this has been disregarded when \( c_i \), was calculated.

(c) Since curves in Fig. 12 were plotted for low-specific-speed pumps where the term \( \frac{w_i^2}{2g} \) is of secondary importance, the cutoff capacity is determined by the impeller-eye velocity \( c_i \) or \( c_o \), and the efficiency point of best efficiency may not coincide with the point where the cutoff capacity occurs. The term \( \frac{w_i^2}{2g} \) is then constant, thus leaving the experimental numerical constant in the right-hand terms of Equations [2] and [3] essentially constant.

With pumps of medium and high specific speed, (1500 to 4000), the cutoff capacity will increase somewhat with the speed if the cutoff takes place to the right of the best-efficiency point. The experimental relationship is the basis of all model testing for cavitation. If the cutoff takes place at partial capacities smaller than normal, Fig. 4, the reason for this is the variation of the coefficient \( \lambda \) in the term \( \frac{w_i^2}{2g} \).

Fig. 17 shows that this is a minimum near the best-efficiency point. When the cutoff takes place at capacities over the normal, the best-efficiency point moves nearer to the cutoff capacity at higher speeds where \( \lambda \) is smaller, and thus \( \frac{w_i^2}{2g} \) is smaller, hence \( c_i^2 \) or pump capacity will increase. At partial capacities, the peak efficiency moves away from the cutoff capacity at higher speeds, while \( \lambda \) is increasing and the cutoff capacity is decreasing.

Looking at the same phenomena from a different point of view, it will be noticed that where the cutoff capacity is nearest the peak efficiency at higher speed, the angle of attack of the incoming flow at the impeller entrance is smaller, the extent of separation is reduced at the vane tips, and the effective area available for the flow is increased. Thus at the same suction pressure, a higher capacity is possible at a higher speed. When the cutoff takes place at partial capacity and the best-efficiency point is moving further at a higher speed, the angle of attack is increasing, separation is more pronounced, the effective area is reduced, and the cutoff capacity is lower at a higher speed in this case.

Since the best angle of attack may not coincide with the point of best efficiency, the effect of the angle of attack on the maximum capacity at several speeds may not be apparent if the cutoff capacity is not sufficiently removed from the best-efficiency point.
(b) Equation [10] holds only at conditions approaching cavitation while the affinity laws still hold. When cavitation sets in, the laws of similarity are not fulfilled and the relationship, \( \sigma = \text{const} \), expressing similarity of conditions as to cavitation becomes approximate only.

(c) Tenot (19) gives the following relationship for the similarity as to cavitation when this has progressed beyond the incipient stage

\[
\frac{\sigma_1 - \sigma_2}{H_1} = \frac{\sigma_3 - \sigma_4}{H_2} \quad \text{........... [11]}
\]

where \( \sigma_1 \) is the critical sigma coefficient which is constant for both model and prototype

\[
\sigma_1 = \frac{\Delta h_1}{H_1} \text{ is sigma for the model}
\]

\[
\sigma_2 = \frac{\Delta h_2}{H_2} \text{ is sigma for the prototype}
\]

Terms \( H_1 \) and \( H_2 \) are the operating heads of the model and prototype, respectively. Tenot has demonstrated the validity of this relationship by high-speed (1/1,000,000 sec) photography of a small propeller pump, operated at several speeds with different suction heads.

Equation [11] can be transformed as follows

Multiply both sides by \( H_1 H_2 \)

\[
\begin{align*}
\frac{\sigma_1 - \sigma_2}{H_1} &= \frac{\sigma_3 - \sigma_4}{H_2} \\
(\sigma_1 - \sigma_2) H_1 &= (\sigma_3 - \sigma_4) H_2 \\
\Delta h_1 - \Delta h_2 &= \Delta h_3 - \Delta h_4 \quad \text{........... [12]}
\end{align*}
\]

Equation [12] shows that for cavitation similarity in two pumps, the absolute pressure at the points of minimum pressure in the impellers is equally removed from the critical pressures (vapor pressure) existing at the incipient cavitation conditions. This means that if two pumps operate at different heads, \( H_1 \neq H_2 \) but the suction pressures are such that \( \sigma_1 = \sigma_2 \), then the pump with the higher head will have cavitation developed to a smaller degree than that prevailing in the low-head pump.

If both model and prototype are tested at the same head \( H_1 = H_2 \), then

\[
\begin{align*}
\frac{\sigma_1 - \sigma_2}{\sigma_3 - \sigma_4} &= 1 \\
\sigma_1 &= \sigma_3
\end{align*}
\]

and, evidently, if Equation [11] holds and \( H_1 \neq H_2 \), \( \sigma_1 \neq \sigma_2 \).

(d) It has been pointed out already that it is difficult to detect the incipient cavitation, and any \( \sigma \) determined as a sigma for the critical cavitation conditions really may represent the state of cavitation progressed sufficiently to be measured by the available testing equipment. Therefore the relationships discussed under (c) are of particular importance.

Again, with wider use of high-specific-speed water turbines and pumps, frequently it becomes uneconomical to provide sufficient submergence to suppress cavitation completely under all operating conditions; therefore unless the heads are reproduced in the model testing the conditions \( \sigma_1 = \sigma_2 \) will only approximately represent the cavitation similarity. In water-turbine practice, when it is impossible to provide a proper submergence due to high cost of excavation, the runner vanes are protected with stainless steel in the places subject to cavitation pitting (20).

(e) To make the discussion of cavitation more definite, the criterion of incipient cavitation should be stated, i.e., whether it is breaking off of the head-capacity curve, drop in efficiency, noise and vibration, or the pitting of the impeller vane. The drop in efficiency is more general as it applies to pumps, irrespective of the specific speed, and may be found while other signs of cavitation are not yet apparent. Depending on the testing facilities and requirements, a drop of 1 per cent or even just a fraction of a point in the efficiency may be taken to indicate that cavitation has already set in.

During cavitation tests, \( \sigma \) variation is obtained either by changing the suction pressure (mostly by throttling), or by changing the pump speed and, hence the head at the same static suction pressure.

While the first method is simpler to arrange, better results are obtained with the variable-head tests. For laboratory testing, a special testing equipment has been used to a limited extent. With this method the pump suction is taken from a vessel which can be kept under different pressures. With a variable-speed drive, this procedure is ideal for accurate \( \sigma \) determination.

Although the same \( \sigma \) value may be obtained with either low head and low suction pressure (high suction lift), or with high head and correspondingly higher suction head (larger pump or higher speed), the physical aspect of the phenomenon as far as cavitation is concerned is not exactly the same. In the first case, the whole suction pipe is under suction lift, and with low velocities, ample time may be available for air or gases to liberate and accumulate in quantities sufficient to impair the pump efficiency and reduce the head capacity before actual formation of vapor bubbles starts. In the second case the pressure drop is mostly dynamic and is limited to a small part of the impeller passages. Besides, with high velocities through the impeller, the time required for the water particles to cross the low-pressure zone is shorter, and, since in all thermodynamic changes time is an essential factor, the relative volume of vaporization and its effect on the pump performance is smaller for high-head pumps (14).

Even for two similar pumps of different sizes operating at the same head and the same \( \sigma \) value (which in this case means the same suction pressure), the extent of cavitation is not in proportion to the pump size, and the bad effects of cavitation will be less pronounced in the large unit.

The water-turbine experience, where model testing is more frequently resorted to than in the pump industry, tends to indicate the truthfulness of the foregoing deductions. F. H. Rogers suggested (21) that "although cavitation starts at the same value of \( \sigma \) for both model and prototype, the vapor-filled cavities are physically about the same dimensions, if the heads are the same, and hence the entire flow pattern through the large runner is affected to a lesser degree than in the case of the small model." Although the velocities at similar points in the impellers are the same in both pumps under such conditions, the effect of the curvature of the impeller profile or suction approach on the velocity distribution (the maximum local velocity) is not the same in the small model and large prototype. The centrifugal forces, which are instrumental in the distortion of the velocity distribution along the curved path, are inversely proportional to the radius of curvature; therefore negotiating the curves through the impeller eye and suction approach in a large pump results in lower maximum local velocities, as compared with the average, than in a small model.

(f) There are several ways to represent graphically the results of cavitation tests. In one of them \( \sigma \) is determined for several points on the head-capacity curve and plotted versus specific speed of the same points. Fig. 20 shows curves for two pumps plotted on this basis. These curves give complete cavitation
characteristics of the pump, irrespective of size and speed.

In another method, efficiency or head is plotted against sigma or suction head, at a constant speed and capacity, the drop in efficiency and head curves indicating the beginning of cavitation, Fig. 14. These curves give cavitation information for one point on the head-capacity curve, and this method is used mostly for model testing when head-capacity conditions are fixed and safe suction head is determined from model testing.

In water-turbine practice, unit capacity and power are plotted versus specific speed. These curves give complete cavitation characteristics for various loads and are independent of head, speed, or size of the unit.

The general trend of sigma variation for the best-efficiency points of pumps of different specific speeds is shown in Fig. 21 and is discussed further under item (i) in this section.

When the plant pumping capacity, head, and suction head are given, the plant $\sigma$ is fixed. By selecting the proper pump speed, pumps of different specific speed may be used to meet the plant requirements with a desired degree of safety against cavitation. Frequently, the speed is also fixed by the specifications. In that case, the specific speed of the plant is fixed. Only a slight variation in pump design is possible in such a case by placing the operating point to the right or left of the best-efficiency point. The rated normal specific speed of the pump at the best-efficiency point will be different from the plant specific speed, but special designs may be resorted to to obtain the desired degree of safety against cavitation. With pumps of high specific speed, where the dynamic depression $\frac{\lambda}{2g}$ plays an important part in setting up cavitation conditions, the number of impeller vanes is an effective means of reducing the critical sigma value without changing the specific speed materially, Fig. 18.

The wide variation of $\sigma$ within the useful range of head capacities is caused partly by the variation of the numerator, representing the dynamic depression, and partly by the variation in the head. In hydraulic turbines where $\sigma$ has been introduced first, the head is essentially constant. When the load on the turbine varies, the variation in $\sigma$ is determined entirely by the pressure conditions at the impeller eye. Thomas (23) suggested using the normal head at the best-efficiency point for all points on the head-capacity curve when calculating $\sigma$ for centrifugal pumps. However, this method has found only limited use in the pump industry.

W. M. White (24) has suggested the use of another factor, "lambda," in addition to sigma. This is defined as

$$\lambda' = H_s + h_r - h_y - \frac{c_i^2}{2g} \tag{13}$$

It will be noted that this is nothing else but the numerator from the expression of Moody's cavitation factor $K_c$, Equation [9], and represents the dynamic depression resulting from the power transmitted by the vanes. It has been pointed out already that the dynamic depression is a predominant factor with the high-specific-speed pumps and varies for different types of pumps along the head-capacity curve. For low-specific-speed pumps, the dynamic depression varies little, therefore $\lambda$, as given by Equation [13], will be independent of the head or specific speed. In this case the cavitation conditions can be predicted from the velocity considerations, and the relationships, given in Fig. 12, may be used.

(i) From theoretical considerations, it is possible to establish a relationship between the $\sigma$ factor and specific speed for best-efficiency points (14, 25)

$$\sigma_2 = \frac{n_2^{1/4}}{n_1} \tag{14}$$

$$\sigma_1 = \frac{n_1}{n_2} \tag{15}$$

The general trend of $\sigma$ variation as function of specific speed as found by actually plotting experimental results agrees very well with Equation [14], as is evidenced, for instance, by the curve published by Wislicenus, Watson, and Karassik, reproduced in Fig. 21, which follows exactly this equation. The scatter of the points about an average curve on the original Wislicenus curve is to be expected, as points were obtained with pumps of different design and were not necessarily located at best-efficiency points. For a series of pumps of consistent design, a continuous curve of $\sigma$ values versus specific speed should be obtained, as all design factors governing cavitation (eye area, number of vanes, etc.) are continuous functions of specific speed. The sigma curve in Fig. 21 can be expressed by the following equations

$$\sigma = \frac{6.3n_2^{1/4}}{10^6} \tag{15}$$

for single-suction pumps and

$$\sigma = \frac{4n_2^{1/4}}{10^6} \tag{16}$$

for double-suction pumps.
(j) Attempts have been made to introduce another cavitation criterion in addition to the generally accepted sigma. It is called "suction specific speed" (26, 27), and is defined as

\[ S = \frac{\text{Rpm} \sqrt{\text{gpm}}}{\Delta H^{1/4}} \]  \hspace{1cm} \text{[17]}

The development of Equation [17] is based on the use of similarity relations (affinity laws), at conditions approaching cavitation and do not establish any new relationship between the variables entering into this expression which cannot be determined from the affinity laws or sigma consideration. There is a fixed connection between the factor \( S \), sigma, and specific speed

\[ \frac{n_s}{S} = \sigma^{1/4} \]  \hspace{1cm} \text{[18]}

Substituting into Equation [18] values of \( \sigma \) from Equations [15] and [16] it is found that

\[ S = 7900 = \text{const for single-stage pumps} \]  \hspace{1cm} \text{[19]}
\[ S = 11,200 = \text{const for double-suction pumps} \]  \hspace{1cm} \text{[20]}

Expressing \( \sigma \) as \( \frac{\Delta H}{H} \) and \( n_s = \frac{\text{Rpm} \sqrt{Q}}{H^{1/4}} \)
Equations [15] and [16] can be transformed to

\[ \Delta h = \frac{(\text{rpm})^{1/4} \cdot Q^{2/5}}{10^4} \times 6.3 \] .......................... [21]

and

\[ \Delta h = \frac{(\text{rpm})^{1/4} \cdot Q^{2/5}}{10^4} \times 4 \] .......................... [22]

respectively. Note that head \( H \) does not appear in Equations [21] and [22], thus indicating that the net positive suction head is independent of the head.

However, this does not mean that the impeller diameter can be cut or extended arbitrarily to obtain any desired head, and expect the relationships, Equations [21] and [22], will hold. For a given head, these equations fix the specific speed and \( \sigma \) values as they appear on curve Fig. 21. This curve applies to pumps of normal design. Evidently if different-specific-speed impellers are obtained by impeller-diameter variation only, the design will not be normal.

(k) When using a model for testing performance and cavitation conditions, the similarity of the model and the prototype should be extended to the suction approach to the impeller and the discharge piping. While this has been fully realized by water-turbine manufacturers, the pump-testing laboratories overlook or underestimate the effects of the suction or discharge piping on the pump performance and behavior as to cavitation.

(f) When applying cavitation data obtained on a small model to the prototype, the suction pressures are usually referred to the impeller-pump center line. However, the points of a minimum pressure may be above this plane of reference, and this distance may be considerably greater on the prototype pump than on the model. Under critical conditions, cavitation may be set up in the larger unit while the model is still free from cavitation.

MEANS TO AVOID OR REDUCE CAVITATION

(a) A knowledge of the cavitation characteristics of pumps is the most important prerequisite of any cavitation-problem study.

(b) Second in importance is the knowledge of existing suction conditions of the plant at the time when the pump selection is made.

(c) An increase of suction-pipe size, reduction of suction-pipe length, elimination of turns, providing a good suction bell, in other words, reduction of losses in the suction pipe, improves the suction conditions of a pump in so far as cavitation is concerned.

(d) An increase in the number of vanes in high-specific-speed pumps, or the removal of parts of the vanes and opening the passages in the impeller eye of low-specific-speed pumps will reduce the minimum suction head to meet fixed head-capacity conditions.

(e) An ample suction-approach area without excessive pre-rotation, a better streamlining of impeller approach, are essential to obtain optimum cavitation characteristics of a pump.

(f) Special materials may be used to reduce the pitting of pump parts due to cavitation, when justified, or if it is impossible to eliminate cavitation by any other means.

(g) The noise and vibration caused by cavitation can be reduced or eliminated by the admission of a small amount of air to the pump suction.

(h) The impeller velocities, impeller-vane load, and head per stage should be low for minimum suction head. All of these factors lead to a bigger pump operated at a low speed, and possibly locating the operating point to the left of the best-efficiency point.

CONCLUSIONS

1. The mechanical nature of cavitation pitting has been established conclusively. Electrolytic and chemical action are of secondary importance or negligible.

2. The suction conditions of the plant should be definitely known when pump selection is made, to avoid cavitation; also, the cavitation characteristics of the pump should be available.

3. The drop in efficiency is the most reliable criterion for detection of cavitation.

4. Cavitation conditions can be predicted for low-specific-speed pumps from the consideration of the velocity through the impeller eye, Fig. 12. For normal design of pumps, \( \sigma \) values can be taken from curve Fig. 21, for any specific speed as an approximation.

5. The law of similarity for cavitation model testing \( (\sigma = \text{const}) \) holds only for conditions approaching cavitation. When cavitation has progressed to some degree, this relationship is approximate only.

6. The harm caused by cavitation is less pronounced in a large pump than in a small model for the same value of \( \sigma \).

7. The presence of gases in liquids does not affect the behavior of pumps as to cavitation, except that vaporization starts at a higher absolute pressure, due to the law of partial pressures. Drop in head capacity may appear earlier on account of liberation of gases of reduced pressure, and the water-hammer effect of collapsing vapor bubbles is cushioned.

8. The life of pump parts can be increased considerably, Figs. 15, 16, by using special materials.

9. Penetration of metals by water under repeated stresses furnishes a logical explanation for the origin of local destructive high pressures found during cavitation, and also in the cases of metal failure by fatigue in presence of liquids.

10. Variation of \( \sigma \) values with specific speed for normal design of pumps can be represented by the equations

\[ \frac{6.3 \cdot n^{1/4}}{10^4} \] for single-suction pumps

and

\[ \frac{1 \cdot n^{1/4}}{10^4} \] for double-suction pumps

Although these relationships have been established experimentally, they have logical theoretical justification.

BIBLIOGRAPHY


Testing of Precision-Lathe Spindles

By G. M. Eoley, Columbus, Ohio

During the war the aircraft industry has demanded the mass production of parts to tolerances previously possible only from the most highly skilled workmanship. In the case of small parts, the tolerances, stated in absolute units, are of the smallest order. This situation led the Bunting Brass and Bronze Company, producer of small bushings, to sponsor research on the design and performance of small precision-lathe spindles, in order that improvements might be made which would result in the production of parts to closer tolerances than had previously been achieved. This paper describes the development and operation of spindle-testing equipment capable of measuring continuously and at any speed changes in the position of the spindle axis relative to the quill as small as one microinch.

The large demand of the aircraft industry for precision parts at the beginning of the war-production period necessitated mass production to tolerances which were formerly met only with the most highly skilled workmanship.

The smallest tolerances, stated in absolute units, are demanded of small parts. The Bunting Brass and Bronze Company, a producer of small bushings, therefore sponsored research on the performance and design of small precision-lathe spindles, mainly with the object of improving the spindle to the point where it would be suitable for production of parts to closer tolerances than have yet been required.

It was immediately apparent that no means existed for measuring the performance of a lathe spindle apart from the machine, and the only test for performance that was in use was the examination and measurement of parts made using the spindle in the lathe or boring mill. It is obvious that the size and shape of such a part may be affected by many factors outside the spindle, yet the source of errors must be known before steps can be taken to remedy them.

Inaccuracies Produced by Poor Spindle Performance

Inaccuracies in the work produced in a lathe originate from "relative" motion between the spindle axis and the work. Instruments which measure vibration of the work or the machine give little information about the spindle, since it is possible that the machine may vibrate as a whole without bad effect on the work, or the source of harmful vibration may not be the spindle.

The spindles tested were to bore smooth round holes or to turn round parts on lathes or boring mills. The quality of such work can be specified in the following terms:

1. Size; inside or outside diameter.
2. Roundness; the approximation obtained to a true cylinder.
3. Straightness; freedom from taper, or the maintenance of required taper.
4. Location of hole or boss.
5. Smoothness of machined surface.

The role of the spindle in determining these qualities is as follows:

1. If the spindle does not rotate always about the same axis there will be a variation in size from place to place on the work.
2. If the axis of rotation varies, but in the same or nearly the same pattern upon each revolution, the work will be out of round.
3. If the spindle rotates first about one axis and then gradually shifts to rotation about another axis, the work will be tapered.
4. If the spindle axis shifts with each new cut, the position of holes bored in a boring mill will vary, or the size of parts turned on a lathe will change.
5. If the spindle axis shifts rapidly and vibrates under cutting loads or from other causes, such as inherent roughness in the spindle, the surface cut will be rough. If the spindle axis shifts noncyclically during the revolution, the surface will also be rough.

The last item is ever-increasing in importance because of the smooth surfaces required on machined parts.

Spindle-Testing Equipment

The testing method adopted was to set up a machine in which the position of the spindle relative to its quill could be measured instantaneously and continuously with a maximum sensitivity better than 1 microinch.

Fig. 1 Spindle-Testing Apparatus

Spindle Support and Drive. The spindle-testing machine is shown in Fig. 1. The spindle itself is lapped into the cradle shown and is held down by straps pulled up by coil springs. The spindle is driven by a 2-hp induction motor through a variable-speed drive (neither of which is visible in the illustration), and the jackshaft shown at the right of the figure.

The entire spindle mount rests on sponge-rubber pads, and the spindle is connected to the jackshaft through a vibration-absorbing coupling. Vibration of the spindle mount during operation is very small; the measuring equipment is, in any case, unaffected by motions of the entire mount.

The spindle can be driven in the machine at speeds up to 3000 rpm.

Measuring Apparatus. The conditions to be met by the measuring equipment were quite severe. It was required to
measure continuously and at any speed the "position" of the spindle axis relative to the quill. The measurement should not disturb the operation of the spindle in any way. The instrument should measure changes of position as small as 1 microinch in order to detect causes of minute roughness. The sensitivity of the apparatus used was actually limited only by the precision with which the reference surface, whose position was measured, could be lapped to a true cylinder.

Among the most critical of these requirements was that the instrument should measure position, not motion; it had to be sensitive to long-period motions, and to hold its zero over relatively long periods. It had, at the same time, to respond without lag to very rapid movements.

The surface whose position is measured is the cylindrical spindle extension at the front end of the spindle in Fig. 1. This was lapped with a ring lap until further lapping produced no change in its shape; it could then be assumed that it was within 5 microinches of a true cylinder.

The measurement made is of the gap between this cylindrical extension and the two "probes" which can be seen fastened to the spindle cradle one on either side of the machine.

The working end of one of these probes is shown in Fig. 2. The two blocks seen in the end are plates of an electrical condenser. The plates are held on a bakelite block, and the gap between them and the outer brass shield of the probe is filled with sulphur. The whole end of the probe was lapped against a cylinder the same diameter as the spindle extension.

The circuit of one of the two measuring channels is shown in Fig. 3. The condenser shown at C1 consists of the plates in the end of the probe together with the spindle extension. The capacitance between the two probe plates varies according to changes in spacing between the spindle extension and the probe.

Condenser C1 is the condenser in the tuned circuit C1-L1 of a push-pull Hartley oscillator. The resonant frequency of such a circuit is

\[ F = \frac{1}{2\pi \sqrt{LC}} \]  

where \( L \) is inductance.

In the tuned circuit L4-C6, the total capacity is composed of a
stray capacitance $C_2$ and the capacitance $C_1$, which is essentially a parallel-plate condenser, in which

$$C_1 = \frac{k}{d}$$

where $k$ is a constant, and $d$ is the spacing between the plates of $C_1$ and the spindle extension.

If $L = L_1$ and $C = C_2 + \frac{k}{d}$ are substituted in Equation [1], differentiation results in the following

$$\frac{df}{dd} = \frac{kf}{2} \left( \frac{1}{kd + C_d^2} \right)$$

The changes in $d$ which it is desired to measure are not more than 1 per cent, so that $df/dd$ is practically inversely proportional to $d$, while on the other hand the sensitivity of the apparatus can be varied at will by larger changes in $d$.

The tuned circuit $L_1-C_1$ was made resonant at about 3200 kc. The output of the oscillator is fed through the buffer amplifier $V_1$ and $V_4$, which prevents movement of the coupling leads and tuning of the amplifier and mixer from affecting the frequency of the oscillator, to the mixer $V_5$. Here the signal is heterodyned with the output of oscillator $V_5$, about 2745 kc, and the approximately 465-kc heterodyne signal is selected by the transformer $T_1$ to be amplified. The voltage of the signal is raised to about 300 volts by the amplifier $V_7$ and amplifier-limiter $V_9$. The signal is then applied to the discriminator tube $V_9$.

The discriminator circuit is one which produces a voltage directly proportional to the difference between the applied frequency and the resonant frequency of the discriminator transformer, $T_2$. The voltage across the terminals marked “to oscilloscope” is thus proportional to small changes in spacing of the condenser $C_1$, which change the frequency of the oscillator $V_1-V_2$ and thus the frequency applied to the discriminator.

Tube $V_9$ was later arranged to operate as a “limiter” amplifier and thus prevented changes in amplification in previous stages from affecting the output voltage.

Each of the two channels is entirely independent. The sensitive condensers are placed 90 deg one to the other, so that each is sensitive mainly to motion along one axis of a system of rectangular co-ordinates. The output of each measuring channel is fed directly to one pair of plates of a cathode-ray oscilloscope tube.

The peak output voltage of the discriminator is about 200 volts, so that no direct-current amplification is necessary. Thus the motion of the cathode beam in the oscilloscope is a magnified representation of the motion of a point at the center of the spindle extension. The magnification most often used was $X \times 10,000$; although magnifications much greater and much less than this are easily available.

It will be apparent that the center of the spindle extension does not in general coincide with the axis of rotation of the spindle. However, if the spindle axis remains the same during the revolution the center of the spindle extension will move in a circle, and the pattern on the oscilloscope screen will be a circle. Deviations from circularity will indicate changes in axis of rotation of the spindle.

Measurements Made

While the design and performance of lathe spindles are outside the scope of this paper, some examples of the measurements made with the instrument are included.

Fig. 4(a) shows the pattern produced on the oscilloscope screen by the rotation of a simple sleeve-bearing precision-lathe spindle, the lubricant being supplied to it by gravity. The magnification of the original, in terms of spindle movement, was $X \times 10,000$, and the pattern shows runout during the revolution of the spindle of about $20 \times 10^{-4}$ in. Fig. 4(b) is the pattern produced by the same spindle when oil was supplied to it at 40 psi. The runout has increased to about $50 \times 10^{-4}$ in., apparently on account of nonuniform oil flow through the spindle.

The reasons for this shift are clearly shown in Fig. 5. Fig. 5(a) is a representation of the contour of the front end of the shaft of this spindle. It was obtained by rotating the shaft of the spindle, noting the deflections of
the galvanometers of the spindle-testing instrument, and plotting the values obtained. It corresponds exactly with the oscilloscope pattern which would be obtained by rotating the shaft at infinitesimal speed. Fig. 5(b) is the contour of the front bearing of the spindle and was obtained by rotating the shaft while a force was applied to it so that the same surface of the shaft would always be in contact with the bearing.

Figs. 5(a) and 5(b) show that the clearance of the spindle may vary by $2 \times 10^{-4}$ in. from point to point around the circumference, and that the place having largest clearance will move during the revolution of the shaft; oil under pressure will thus increase the axial shifts of this particular spindle.

In order to study the effect of a vibrating load on the spindle, similar to that which would occur in cutting, a small ball bearing was placed on the spindle extension, the inner race of which moved with the spindle while the outer race was held still by the force of gravity on a small weight attached to it. Fig. 6 shows the oscilloscope patterns produced under such conditions in the plain-bearing spindle both with gravity oil feed and pressure feed. It will be seen that the rigidity of the spindle under such loading is not much affected by oil pressure.

The manner in which the spindle-testing equipment can be used to measure long-time changes in spindle position is shown in Fig. 7. Fig. 7(a) is a double exposure of the oscilloscope patterns before and after a 5-min shutdown of the spindle. The spindle axis has changed about $10^{-4}$ in. relative to the probes on account of cooling of the machine. A similar double exposure before and after a 1-hr shutdown, Fig. 7(b), shows that a shift of about $3 \times 10^{-4}$ in. has occurred in this time. This shift, or a greater one, might be cause for rejection of some parts made on a machine after such a shutdown if the spindle had this performance.

Fig. 8 shows the pattern produced during two successive revolutions of a spindle containing a pair of superprecision combination radial-and-thrust ball bearings in the back end and a plain bearing in the front end. Unlike the plain-bearing spindle, the axis of this spindle does not retrace its path during successive revolutions on account of the ball-bearing races rotating at about one half the rate of the spindle. The precession of the axis of this spindle has an amplitude somewhat greater than 30 microradians per revolution, sufficient to produce a roughness quite intolerable on some machined parts.

**Conclusion**

The apparatus described was found very useful in providing information for use in the design of new precision-lathe spindles, and in checking the performance of them after they were built. It is apparent that the same type of circuit may be useful in other cases where measurements of very small changes in position or size are to be made. While the advantages of this design over some other electrical micrometers and strain gages are most apparent where the demand is for an instrument to measure displacements both at very high and zero rates of change, the way in which the frequency-modulation principles used free the equipment from errors caused by changes in amplification is also desirable in other applications.

*(Owing to travel emergency conditions existing when this paper was presented, written discussion will be accepted until November 10, 1945.)*
Irreversibility in the Theoretical Regenerative Steam Cycle

BY R. E. HANSEN, NEW YORK, N. Y.

An irreversible process takes place in the theoretical regenerative steam cycle for power generation when superheat in the steam bled from the turbine is transferred to feedwater. Increase in entropy occurs, as in any irreversible process; this increase can be determined by a simple method of graphic integration, and used in computing additional heat rejection to the condenser. Heat rate of the cycle can then be found with a high degree of accuracy by a simple formula.

NOMENCLATURE
The following nomenclature is used in the paper:

\( H \) = enthalpy of steam at point indicated by subscript, Btu per lb

\( h \) = enthalpy of water at point indicated by subscript, Btu per lb

\( dp \) = increase in pressure between adjacent infinitesimal feedwater heaters

\( s \) = entropy per pound of steam at point indicated by subscript, Btu per deg F

\( S \) = entropy when used to indicate total in system, Btu per deg F

\( \Delta S \) = total gain in entropy for cycle with 1 lb of steam at throttle, Btu per deg F

\( T \) = absolute temperature at point indicated by subscript, deg F

\( v \) = specific volume of feedwater in heater

\( W_B \) = work lost during an irreversible process

\( dW \) = fraction of throttle steam bled at any stage

\( 1 - w \) = quantity of feedwater in any heater, with steam quantity at throttle taken as unity

INTRODUCTION
Analyses of the theoretical regenerative steam cycle have been made in the past, with the purpose of developing procedures for computing heat rate. Methods that have heretofore been presented, however, are laborious and leave much to be desired in the way of simplicity in use. The purpose of the present paper is to show that by computing the increase in entropy which occurs in the cycle, the heat rate may be determined quickly and with a high degree of accuracy.

The theoretical regenerative cycle is one in which steam is expanded adiabatically; a sufficient quantity is bled from the turbine at an infinite number of points to heat feedwater in an infinite number of open (contact) heaters to the temperature at which evaporation occurs. Feed-pump work is done at 100 per cent efficiency. Boiler-plant, pipe-friction, radiation, and generator losses, also temperature differences in conduction, are considered zero, the limit they would approach if size of equipment and insulation were indefinitely large.

The provisions as set forth require that all processes in the cycle be accomplished reversibly except one. Steam bled from the turbine in the superheat region is mixed irreversibly with feedwater, the latter being at the same pressure as the steam but at its saturation temperature. During this irreversible process, an increase in entropy occurs. Usually, engineers are reluctant to utilize the concept of entropy except when it remains constant, but, as will be shown, the quantity is a convenient device, even when it is a variable.

EQUIVALENCE OF REVERSIBLE CYCLES
Steam can theoretically be made to do work in the cycle ABCDEA, Fig. 1, condensation being stopped at point E, and wet steam being compressed adiabatically to point A. In the regenerative cycle, steam is completely condensed to point G on the liquid line, steam being bled from several stages in the turbine to heat condensate to point A. If this process of regeneration were strictly reversible, the cycle would then be exactly equivalent to cycle ABCDEA, the greater quantity of heat rejected to condensing water per pound of steam going to the condenser being exactly compensated by a decrease in the quantity of steam condensed. This is a consequence of Carnot's law that each increment of heat added in a cycle at absolute temperature \( T_i \) and rejected at temperature \( T_j \) is utilized with efficiency \( (T_i - T_j)/T_i \), provided no irreversible processes occur within the cycle. Any cycle in which the pattern of heat input is along line ABC has the same efficiency as the cycle ABCDEA, provided only that all heat rejection is at the same temperature and that no irreversible processes are used.

CYCLES WITH VARYING STEAM QUANTITY
To assist in visualising the cycle, the diagram may be con-
sidered to have a third dimension, i.e., quantity of steam. Any horizontal cross-sectional area of the resulting solid would then represent a total, rather than a unit, quantity of entropy. Rankine cycle, shown by \( ABCDGA \), has a constant steam quantity, hence front and rear faces of the solid representing it are plane and parallel, and all vertical sections parallel to the temperature-entropy plane are identical. In the theoretical regenerative cycle, the weight of steam in the turbine diminishes as expansion proceeds from \( C \) to \( D \), Fig. 1, and the weight of feedwater increases as it is heated from \( G \) to \( A \). The total quantity of entropy represented by any horizontal cross section in Fig. 2 would remain constant below the plane of saturation temperature at the throttle, if a completely reversible regenerative cycle were used, as illustrated by solid lines in Fig. 2. Entropy may here be regarded as transferred from bled steam to feedwater, without increase in total quantity in the system.

### Computing Entropy Increase

To determine the increase in entropy resulting from the irreversible process, the first law of thermodynamics is used. Thus if a quantity of heat, represented by an area \( T_1S \) on the temperature-entropy diagram, Fig. 3, is cooled without doing work to temperature \( T_2 \), then the following relation must hold

\[
T_1S = T_2(S + \Delta S)
\]

from which

\[
\Delta S = \frac{S(T_1 - T_2)}{T_2} = \frac{W_R}{T_2}
\]

where \( W_R \) is the quantity of work that could have been generated in a reversible cycle. This may also be expressed differentially, as

\[
dS = \frac{dW_R}{T_2}
\]

To be reversible, the process of regeneration must be accomplished in such a way that heat passes from one fluid to another at the same temperature. As long as wet steam is bled in infinitesimal steps and gives up its heat in feedwater heaters each operating with zero terminal difference, the process is fully reversible. But when superheated steam is bled and used at constant pressure in the heaters, part of the process is not reversible. This is because heat at temperature higher than saturation passes to water at saturation temperature. To accomplish reversibility in the superheat region, it would be necessary to compress bled steam isothermally to saturation pressure corresponding to its superheat temperature, as from \( Y \) to \( Z \) in Fig. 1, utilizing the heat rejected in isothermal compression for heating the feedwater. If instead steam is bled at point \( X \) where the expansion process has progressed to a pressure equal to that of the saturated steam at feedwater temperature and is cooled irreversibly, work represented by triangle \( XYZ \) is lost. Then, from Equation [1]

\[
dS = \left[ H_x - h_f - T_f(s_x - s_f) \right] \frac{d\omega}{T_f}
\]

if \( d\omega \) is the fraction of the throttle steam bled at any stage and subscript \( f \) refers to the condition of saturated liquid at the pressure of the bled steam.

The heat balance for the infinitesimal heater is

\[
d\omega(H_x - h_f) = (1 - \omega) [dh_f - \rho dp] = (1 - \omega) T_f ds_f \quad \text{[3]}
\]


\[
dS = \left[ \frac{H_x - h_f - T_f(s_x - s_f)}{H_x - h_f} \right] (1 - \omega) ds_f \quad \text{[4]}
\]

The quantity of steam in the turbine at any temperature \((1 - w)\) is found from the cross-sectional area of the three-dimensional diagram shown in Fig. 2. As stated before, this area remains constant except for the accumulated increase in entropy occurring as regenerative heating proceeds. Such cumulative increase is zero at the throttle, where its rate of increase is greatest, and reaches its maximum value when bled steam becomes saturated, at which point no further increase occurs. It is therefore sufficiently accurate for the purpose of evaluating \(1 - w\) to assume the cross-sectional area constant, that is

\[
(1 - w) \times (S_X - S_f) = 1 \times (s_C - s_A)\ldots \ldots [5]
\]

The magnitude of inaccuracy resulting from this simplifying assumption is about 1 per cent of \(\Delta S\) at \(3200\) psia \(1200\) F and much less at lower throttle conditions. The corresponding error in heat rate, computed as will be shown, is less than 2 Btu per kwhr. Making this substitution and simplifying produces

\[
dS = (s_C - s_A) \left[ \frac{1}{s_X - s_f} - \frac{T_f}{H_X - h_f} \right] ds_f \ldots \ldots [6]
\]

Equation [6] is integrated between limits of \(s_f\) corresponding to the throttle pressure and the point where the expansion curve crosses the saturation line by plotting the values of the bracketed coefficient against \(s_f\) as in Fig. 4, and multiplying the area under the curve by the constant term \((s_C - s_A)\); this product is \(\Delta S\). A sample calculation is shown in Table 1 for throttle conditions 3200 psia 1200 F, with \(s_X\) equal to 1.5745. Values from column 10 are plotted in Fig. 3, and the area, computed as 0.0288, is multiplied by \(s_X - s_A = 0.542\); the product is 0.0156, which is the value of \(\Delta S\) for use in Equation [7].

### Table 1: Computation of Expression in Equation [6] for \(s_X = 1.5745\), Corresponding to 3200 Psia 1200 F

<table>
<thead>
<tr>
<th>Psia</th>
<th>(s_f)</th>
<th>(s_X - (s_f - 1))</th>
<th>(T_f)</th>
<th>(h_f)</th>
<th>(H_X)</th>
<th>(h_f)</th>
<th>(s_C - s_A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3200</td>
<td>0.8320</td>
<td>0.5425</td>
<td>1.843</td>
<td>1164.8</td>
<td>1569.9</td>
<td>572.4</td>
<td>697.5</td>
</tr>
<tr>
<td>2800</td>
<td>0.945</td>
<td>0.6286</td>
<td>1.591</td>
<td>1144.7</td>
<td>1548.0</td>
<td>770.1</td>
<td>777.9</td>
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<tr>
<td>2400</td>
<td>0.6923</td>
<td>0.6722</td>
<td>1.487</td>
<td>1121.8</td>
<td>1523.6</td>
<td>718.4</td>
<td>805.2</td>
</tr>
<tr>
<td>2000</td>
<td>0.6819</td>
<td>0.7156</td>
<td>1.403</td>
<td>1066.5</td>
<td>1485.6</td>
<td>671.7</td>
<td>833.6</td>
</tr>
<tr>
<td>1600</td>
<td>0.8196</td>
<td>0.7549</td>
<td>1.323</td>
<td>1004.6</td>
<td>1462.8</td>
<td>624.1</td>
<td>858.7</td>
</tr>
<tr>
<td>1200</td>
<td>0.7711</td>
<td>0.8034</td>
<td>1.245</td>
<td>938.9</td>
<td>1422.8</td>
<td>577.1</td>
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<tr>
<td>800</td>
<td>0.7108</td>
<td>0.8637</td>
<td>1.158</td>
<td>877.9</td>
<td>1370.6</td>
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<td>400</td>
<td>0.6214</td>
<td>0.9331</td>
<td>1.049</td>
<td>804.3</td>
<td>1291.3</td>
<td>414.0</td>
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<tr>
<td>141</td>
<td>0.5076</td>
<td>1.0669</td>
<td>0.967</td>
<td>731.3</td>
<td>1193.1</td>
<td>325.4</td>
<td>807.7</td>
</tr>
</tbody>
</table>

### Table 2: Computed Heat Rates Compared with Published Data

<table>
<thead>
<tr>
<th>Pressure</th>
<th>Temp. deg F</th>
<th>Condenser pressure, In. Hg abs</th>
<th>Value of (\Delta S)</th>
<th>Heat Rate, Btu per kwhr</th>
</tr>
</thead>
<tbody>
<tr>
<td>3200</td>
<td>1200</td>
<td>1.0</td>
<td>0.0156</td>
<td>5999</td>
</tr>
<tr>
<td>850</td>
<td>900</td>
<td>1.0</td>
<td>0.0036</td>
<td>7255</td>
</tr>
<tr>
<td>450</td>
<td>700</td>
<td>1.0</td>
<td>0.0014</td>
<td>8229</td>
</tr>
</tbody>
</table>

Heat rate is given as follows:

\[
\text{Theoretical heat rate} = \frac{3412.75 \cdot (H_C - h_f)}{H_X - h_f - T_v \cdot (s_C - s_A + S_f)}\ldots \ldots [7]
\]

If \(\Delta S\) is determined accurately, Equation [7] is exact.

In a paper\(^3\) presented last year, Messrs. Selvey and Knowlton gave heat rates accurately computed for a cycle as defined in the previous paper; the nature of the computations is such that slide-rule accuracy will usually be adequate.

2. The integration for \(\Delta S\) may be performed between different limits on a single curve, such as that in Fig. 4, so that the computation of a few such curves will suffice to give the complete range of values for all commonly used pressures and temperatures. The quantity \(\Delta S\) being small compared with \(s_C - s_A\) can be obtained with sufficient accuracy by interpolation even though the intervals are quite large.

3. The basis of derivation is concerned only with the fundamental properties of steam and eliminates the need for dealing separately with the work done by the boiler feed pump; attention thus being focused on the essential and limiting factors.

4. Data can be more readily utilized in working with complex cycles in which steam is reheated after partial expansion. In

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this case $\Delta S$ for partial expansion would be that for the throttle condition minus that for the uncompleted part of the expansion and is added arithmetically to $\Delta S$ for additional expansions, each multiplied by the fraction of the original steam remaining at the beginning of expansion.

5 $\Delta S$ is independent of back pressure, unless the exhaust is superheated; no back-pressure corrections need be computed.

A complete table of heat rates for the theoretical regenerative cycle having already been published, \(t\)he author has not undertaken the computation of exact values of $\Delta S$ at this time. Usefulness of such compilation would be largely for special cases; when needed, $\Delta S$ can be computed from published theoretical heat rate by means of Equation [7], or by the method just given. To show approximately how quantity varies, however, Fig. 5 is presented. It is based on rough calculations covering pressures up to 3000 psia and temperatures 600 to 1600 F.

**Nonadiabatic Turbine Expansion**

In applying the method as described to the determination of cycle performance with an actual turbine, the entropy increase due to irreversibility in feedwater heating becomes greater than with the cycle as previously defined. The higher superheat at intermediate bleed points is responsible for this condition, which can be taken accurately into account only by recomputing $\Delta S$, using the actual expansion curve. An additional $\Delta S$ value because of irreversibility in turbine expansion can be read from the expansion curve, and the sum of these used to determine the heat rate of another cycle, also having an infinite number of bleed points.

The result under the method indicated would not be the same as obtained by dividing the heat rate of the theoretical adiabatic cycle by the over-all turbine efficiency, though the difference may be small. It follows that whatever method is used for determining the heat rate of the adiabatic cycle, the entire calculation must be repeated for the nonadiabatic cycle, if exact results are needed.

*(Owing to travel emergency conditions existing when this paper was presented, discussion will be accepted until November 10, 1945)*
Developments in boxcar construction are traced through the period of the 1920's and early 1930's, leading to design work by K. F. Nyström on the Milwaukee Road in which 15-gage side sheets, stiffened by six longitudinals of the lapped type formed integrally with the side sheet, were found to be entirely satisfactory for car-side construction, and accomplished a saving in weight of 600 lb. In order to determine the possibility of further weight reduction without sacrificing strength, a study was made of the elastic stability of various types of car sides, and tests were conducted to correlate the theoretical study with actual test results. This paper includes a discussion of shearing stress in thin-webbed girders, the test procedure, a theoretical analysis of a number of panels tested, and the results obtained for the panels tested.

INTRODUCTION

During the late 1920's and early 1930's the trend in boxcar construction was directed from single-sheathed, diagonal-braced construction to the steel-sheathed, wood-lined type of car. The early frame construction was of the truss type and the sheathing was not employed as a part of the load-carrying structure. The steel-sheathed wood-lined cars did not utilize diagonal bracing, and the car structure was therefore similar to a plate girder, the steel side sheets serving as the web of the girder. In this construction the high shear loads at the ends of the beam are transmitted to the supports by the web working in shear. The early steel-sheathed cars utilized rather heavy-gage steel side sheets approximately 0.100 in. thick, and such cars weighed approximately 46,500 lb for cars 40 ft 6 in. in length.

Prior to 1925 the load limit on boxcars was determined on the basis of 110 per cent of the nominal capacity. This method of determining the load limit fixed the maximum lightweight for a 40-ton freight car at 48,000 lb, and for a 50-ton freight car at 59,000 lb. Since the load-carrying capacity of the car was thus definitely determined there was no great advantage in reducing the weight below this maximum light weight. However, in 1925 the procedure for calculating the load limit was modified. The load limit was to be determined by obtaining the difference between the permissible weight at the rail, based on journal size, and the lightweight of the car. Thus weight reduction became of prime importance, as every pound removed from the car structure could be replaced by a pound of tariff-producing lading. Since the weight of the side sheets on a 40-ft 6-in. boxcar with 0.100-in. side sheets was approximately 2735 lb, decreasing the thickness of these sheets was a fertile field for weight reduction, and some boxcars were built with 14-gage (0.075 in.) side sheets stiffened with a very few heavy stiffeners. The weight of the side sheets on a 40-ft 6-in. boxcar of this type was approximately 2350 lb, a desirable weight reduction. These cars did not collapse and are still in service, but the side sheets developed stress buckles at the bolster panels under heavy loads.

It was decided, nevertheless, that the 14-gage sheets were, if adequately stiffened, suitable for side sheets and K. F. Nyström of the Milwaukee Road developed a car side utilizing 14-gage sheets with six longitudinal stiffeners of the lapped type formed integrally with the side sheet as shown in Fig. 1. A large number of cars were constructed with this type of side with completely successful results, as the side sheets showed no evidence of the formation of buckles under load. The weight of the side sheets on a 40-ft 6-in. boxcar of this type was 2495 lb, which is 145 lb more than that of the previously discussed inadequately stiffened 14-gage side construction, but is 240 lb less than the weight of the side sheets on a car with 0.100-in. side sheets.

In order to determine the possibility of accomplishing further weight reduction without sacrificing strength a study was made of the elastic stability of various types of car sides, and a number of tests were conducted to correlate the theoretical study with actual test results. The following types of side construction were considered in the theoretical analysis:

1. A flat, unstiffened side section with 15-gage side sheets.
2. A side section consisting of 15-gage side sheets stiffened with two lapped longitudinal ribs.
3. Same side section as No. 2 with a 20-gage vertical stiffener added.
4. The car-side section previously used on the Milwaukee Road, consisting of a 14-gage side sheet with six lapped, longitudinal stiffeners.
5. A side section similar to No. 4 with 14-gage side sheets and six lapped longitudinal stiffeners smaller than those used for No. 4.
8. A flat unstiffened side section with 0.100-in. side sheets.

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1 Chicago, Milwaukee, St. Paul & Pacific Railroad.
2 Milwaukee, Wis. Contributed by the Railroad Division and presented at the Annual Meeting, New York, N.Y., Nov. 27-Dec. 1, 1944, of the American Society of Mechanical Engineers.
3 Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.
Test panels representative of side sections 1 to 5, inclusive, were fabricated and tested to check the analytical results.

This paper will include a discussion of shearing stresses in thin-webbed girders, the test procedure, a theoretical analysis of each panel, and the test results for the panels tested.

Shearing Stress in Thin-Webbed Girders

The web of a plate girder is a thin plate subjected to shear, compressive, and tensile stresses. The shear and compressive forces tend to buckle the web of the girder.

The plate girders utilized on railway and highway bridges have webs which are thick enough to resist this buckling or have web stiffeners which prevent it. In this type of construction the high shear loads at the ends of the beam and concentrated loads are transmitted to the supports by the web working in shear. The sides of boxcars with heavy side sheets or stiffened thin side sheets are girders of this type.

Other plate girders, for example, those utilized in airplane construction, are not designed to carry the entire load to the support by the shear in the web. A part of the load is carried as shear in the web, and the additional load causes the formation of buckles which permit the web plate to work as a diagonal tension tie. Any stiffeners then act as struts and the load is transmitted as in a truss. The load at which the flat form of equilibrium becomes unstable and the plate begins to buckle is known as the critical load.

The load at which buckle formation begins is dependent on the web thickness and the stiffener strength and spacing. The critical shear or compressive load is proportional to the flexural rigidity of the plate. The resistance to buckling can therefore always be increased by increasing the thickness of the plate, but such a design will not be economical in respect to the weight of material used in deep girders such as boxcar sides. It is generally more economical to keep the side sheets as thin as possible and provide the necessary stability by utilizing posts and longitudinal lapped ribs at stiffeners. The weight of the longitudinal ribs will usually be much less than the additional weight required to provide a plate of adequate thickness.

All boxcar sides must have vertical stiffeners in the form of posts to resist bulging and for other practical reasons. If longitudinal ribs are also used in conjunction with the posts, they must be so located and of such dimensions to prevent buckling of the web. When posts only are used for stiffeners it is possible for buckles to form, allowing the girder to work as a truss as previously mentioned, but if adequate longitudinal ribs are introduced the buckle cannot pass the rib without bending it. Thus the girder has been broken up into rectangular panels bounded by the posts and longitudinal ribs. If the reinforcements are of the lapped-rib variety, it is possible to get the advantage of longitudinal stiffeners and at the same time have the safety factor of being able to go into a tension field if for some reason the unit buckling stress is exceeded. In designing a girder using thin flat sheets or thin sheets stiffened with lapped ribs, a low factor of safety in the neighborhood of 1.5 to 1.25 can be used because buckling of the sheets does not mean immediate failure of the structure since the sheet can go into a diagonal tension field. This will be accompanied by excessive deflection, but the structure will not collapse.

The determination of the spacing of stiffeners, and the size of stiffeners, for a given thickness of web must be based on a study of the elastic stability of the web plate. An excellent discussion of the rational theory of buckling of thin plates is given by S. Timoshenko, and the following theory is credited to him.

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G R E E N , D R IN K A — C R ITIC A L SHEARING STRESS IN BOXCAR SIDES

F i g . 4 L im i t i n g V a l u e s o f R a t i o \gamma f o r C a s e o f O n e K in

V is Poisson's ratio and b and d are the dimensions of the panel as indicated in Fig. 2.

T e s t P r o c e d u r e

All of the test panels were fabricated with identical posts, side plates, and side-sill upper elements. The panels tested were subjected to a shear load as shown in Fig. 6, applied with a geared jack, and measured with a 0 to 50,000-lb dynamometer. The panels were loaded until attempts to apply additional load resulted only in additional strain of the test panel.

The experimental determination of the exact critical shearing load of a sheet or plate is rather difficult. Theoretically, the critical load is the load at which the flat form of equilibrium becomes unstable and the plate begins to buckle. The exact load at which this occurs is not easily determined. To facilitate the determination of the critical shearing loads of the panels tested the side sheets were carefully observed, and it was noted that in the cases where the panels failed due to buckling of the side sheets, the load was taken by the panels with some lateral movement of the sheet, such as straightening of buckles initially present due to welding, and then a load was reached at which buckles with a definite pattern were formed. The load at which the latter occurred was considered the critical shearing load.

Panel No. 1. This test panel, which consisted of a long narrow 15-gage sheet, stiffened only by the posts, is shown in Fig. 6, the posts being stiff enough to resist bending when the sheet buckled. The critical shearing stress can be obtained by considering a panel as shown in Fig. 2, with b = 28 in., and d = 113 1/8 in. Applying Equation [1], the critical shearing stress is 870 psi. The effective shear area, consisting of the vertical portion of side plate, the vertical portion of the upper element, and side sheet, is 10.63 sq in. Thus the critical shearing load is

\[ Q_{cr} = 10.63 \times 870 = 9250 \text{ lb} \]

The actual test critical shear load of this panel was 12,000 lb.

Panel No. 2. This test panel, consisting of a side section with 15-gage side sheets, and two lapped, longitudinal ribs, is shown in Fig. 7. The flexural rigidity of 15-gage sheet is 827 lb-in.

This test section can be considered as having two ribs dividing the plate into panels with dimensions \( a/d \) and \( b \), where \( a = 113 1/8 \) in. and \( b = 28 \) in. Thus \( a/b = 4.06 \). From Fig. 5, \( \gamma = 0.37 \). Thus the required rib flexural rigidity is

\[ B = 0.37 \, Da = 0.37 (827 \times 113.625) = 34,750 \text{ lb-sq in.} \]
From Fig. 5 it can be seen that the value of $\gamma$ is even less than that for test No. 2 which did not utilize a vertical stiffener. Thus the required rib flexural rigidity is less than 34,750 lb-sq in. and since the flexural rigidity of the actual longitudinal rib, Fig. 8, is 466,000 lb-sq in., it is apparent that the longitudinal rib is sufficiently rigid. Therefore the sheet will buckle without bending the longitudinal ribs.

In order to determine the flexural rigidity of the vertical stiffener required to prevent bending of the stiffener when the plate buckles, a unit panel with one stiffener can be considered as shown in Fig. 3, with $a = 28$ in. and $b = 37\frac{1}{2}$ in. Thus the ratio $a/b$ is 0.747. From Fig. 4, $\gamma$ is 30. This value of $\gamma$ was taken from the broken-line portion in Fig. 4. The dotted portion of this curve is merely an extension of the data given by Timoshenko and should be accepted only as an approximation. Considering $\gamma$ as 30, the required vertical-stiffener flexural rigidity is

$$B = 30 \times \frac{D_a}{\gamma} = 30 \times 827 \times 28 = 695,000 \text{ lb-sq in.}$$

A cross section of the vertical stiffener is given in Fig. 10. Since the flexural rigidity of this vertical stiffener is only 371,000 lb-sq in. it is apparent that the vertical stiffener will bend when the sheet buckles.

In order to approximate the critical shearing load for this panel, the critical shearing load of a panel divided as shown for the test panel, but with a vertical stiffener of sufficient rigidity, will be considered. This hypothetical panel can be considered as divided into panels such as that shown in Fig. 2, with $b = 14$ in. and $d = 37\frac{1}{2}$ in. For such a panel the critical shearing stress is, by Equation [1], 3685 psi. The effective shear area is the same as that for test panel No. 2; that is, 11.14 sq in. The critical shearing load is therefore

$$Q_{cr} = 11.14 \times 3685 = 41,100 \text{ lb}$$

However, as previously stated, the vertical stiffener used for this test was not rigid enough to resist bending when the sheet buckles. Failure would therefore occur at some load between the critical shearing load obtained for panel No. 2, which consisted of the same panel without a vertical stiffener, and the critical shearing load of the hypothetical panel discussed which had a sufficiently rigid vertical stiffener. It will be assumed that the additional load-carrying capacity due to the application of the vertical stiffener is in proportion to the ratio of the flexural rigidity of the rib to the required rigidity of the rib. Thus on the basis of this crude assumption, the critical shearing load for panel No. 3 is

$$Q_{cr} = 13,150 + \frac{371,000}{695,000} \times (41,100 - 13,150) = 28,070 \text{ lb}$$

The actual test critical shear load of this panel was 36,000 lb.

Panel No. 4. This test panel, utilizing 14-gage side sheets and
six longitudinal lapped stiffeners, is shown in Fig. 11. This side section was used on a large number of cars constructed by the Milwaukee Road.

The flexural rigidity of 14-gage (0.075-in.) sheet is 1160 lb-in. To determine the required longitudinal rib flexural rigidity to cause the panels bounded by the posts and longitudinal ribs to buckle without bending the ribs, a unit panel with one stiffener, Fig. 3, with $a = 31\frac{1}{8}$ in. and $b = 28$ in., can be considered. The ratio $a/b$ is 1.133 and from Fig. 4, $\gamma$ is 9.15

The required rib flexural rigidity is therefore

$$B = 9.15 \times Da = 9.15 \times 1160 \times 31.75 = 337,000 \text{ lb-sq in.}$$

If a unit panel, having two stiffeners, is considered having $a = 47\frac{1}{8}$ in. and $b = 28$ in., the ratio $a/b$ is 1.703, and from Fig. 5, $\gamma$ is 6.80.

The required rib flexural rigidity is therefore

$$B = 6.80 \times Da = 6.80 \times 1160 \times 47.625 = 376,000 \text{ lb-sq in.}$$

Thus on the basis of a unit section with one stiffener, the required stiffener flexural rigidity is 337,000 lb-sq in. While for a unit section with two stiffeners, the required stiffener flexural rigidity is 376,000 lb-sq in. It can be seen that the required rib rigidity increases slightly as the number of stiffeners increases. Timoshenko does not extend the theory regarding the required rib rigidity beyond the case of a panel with two stiffeners. However, for practical cases it will be assumed that the required stiffener flexural rigidity will not be larger than 1.5 times that for a panel with one stiffener. Thus a well-proportioned rib for this test panel should have a rigidity of approximately

$$B = 1.5 \times 337,000 = 505,500 \text{ lb-sq in.}$$

A cross section of the actual rib used on this test panel is given in Fig. 1. The flexural rigidity of this stiffener is 680,000 lb-sq in. The stiffener is therefore conservatively designed, and it is apparent that the panel would fail due to buckling of the sheet in panels with $b = 15\frac{1}{8}$ in. and $d = 28$ in., Fig. 2. Thus the critical shearing stress is, by Equation [1], 4020 psi. The effective shearing area, consisting of the vertical portion of side plate, the vertical portion of upper element, and the side sheets, is 13.86 sq in. and the critical shearing load is therefore

$$Q_{cr} = 13.86 \times 4020 = 53,700 \text{ lb}$$

The actual test critical shear load of this panel was not determined as the posts buckled due to compression at 43,000 lb without buckling the side sheets.

Panel No. 5. This test panel, utilizing 14-gage side sheets, is shown in Fig. 11. The panel is stiffened with six lapped longitudinal ribs smaller than those employed for panel No. 4. Since the spacing of posts and longitudinal ribs is the same for this panel as for panel No. 4, the required longitudinal rib flexural rigidity based on a unit panel with one stiffener is the same as that for panel No. 4, that is, 337,000 lb-sq in. As stated in the discussion of test No. 4 for practical cases with several stiffeners, it is advisable to make the stiffener about 1.5 times the rigidity required on the basis of a calculation for a unit panel with one stiffener, that is, 505,500 lb-sq in. for this panel.

A cross section of the lapped longitudinal stiffener used for this test is given in Fig. 8. The flexural rigidity is 510,000 lb-sq in. This stiffener is therefore well designed, and it is apparent that the longitudinal rib will not bend when the sheet buckles. The sheet will therefore buckle in panels bounded by the posts and longitudinal ribs with $b = 15\frac{1}{8}$ in. and $d = 28$ in., Fig. 2. This is the same as for panel No. 4, and the critical shearing stress based on Equation [1] is again 4020 psi. The effective shearing area, consisting of the vertical portion of side plate, the vertical portion of upper element, and the side sheets, is 13.20 sq in. Thus the critical shearing load is

$$Q_{cr} = 13.20 \times 4020 = 53,000 \text{ lb}$$

The actual test critical shear load of this panel was not determined as the posts buckled due to compression at 43,000 lb without buckling the side sheets.

Panel No. 6. This test section, Fig. 11, with six lapped longitudinal stiffeners of the cross section shown in Fig. 12, was used on 1000 boxcars recently built at the Milwaukee shops. The side sheets were of 15-gage thickness.
This panel was not tested.

Panel No. 7. This side section is exactly the same as panel No. 6 with the exception that the side sheets were 16 gauge. The required longitudinal stiffener flexural rigidity can again be determined on the basis of a unit panel with one stiffener, Fig. 3, with \( a = 31\frac{3}{4} \text{ in.} \) and \( b = 28 \text{ in.} \). Thus the ratio of \( a/b = 1.133 \), and from Fig. 4, \( y = 9.15 \). Since the flexural rigidity of 16 gauge sheet is 594 lb-in., the required longitudinal flexural rigidity for a unit panel with one stiffener is

\[
B = 9.15 \times Da = 9.15 (594 \times 31.75) = 172,800 \text{ lb-sq in.}
\]

As previously stated, the flexural rigidity of a stiffener for a panel with a multiple number of stiffeners should be approximately 1.5 times that for a unit panel with one stiffener. Thus for this panel the longitudinal stiffener should have a flexural rigidity of 259,200 lb-sq in. The flexural rigidity of the 16-gage stiffener shown in Fig. 12 is 346,500 lb-sq in. Thus it is apparent that the sheet will buckle in panels with \( b = 157/6 \text{ in.} \) and \( d = 28 \text{ in.} \). From Fig. 4, \( y = 9.15 \). Since the flexural rigidity of 16-gage sheet is 594 lb-in., the required longitudinal flexural rigidity for a unit panel with one stiffener is

\[
B = 9.15 \times Da = 9.15 (594 \times 31.75) = 172,800 \text{ lb-sq in.}
\]

As previously stated, all of the panels tested were loaded until attempts to apply additional load resulted only in additional strain of the test panel. This additional strain consisted of raising higher sheet buckles in tests Nos. 1 and 2; raising higher sheet buckles and bending vertical stiffeners in test No. 3, and buckling the posts in tests Nos. 4 and 5.

Conclusions

In an actual car side a great deal more load beyond the critical shearing load can be carried by development of tension fields. Such a tension field is, of course, accompanied by extremely high deflections. These high deflections result in rotation of the doorpost by the side sill and development of high fixed end moments which often result in failure of the doorposts or doorpost attachments in cars with unstiffened side sheets.

The panels tested in tests Nos. 4 and 5 failed at 43,000 lb, due to compressive buckling of the side posts; in actual car construction the shear load is transmitted to the bolster by two posts, and hence a load of 86,000 lb would be required to buckle the posts. Since the actual maximum shear load at the bolster of a 50-ton box or automobile car is 21,000 to 22,000 lb, the post-buckling factor of safety is approximately 4.

The test and analytical results are in reasonably good agreement. The analytical critical shearing loads for the panels which failed due to buckling of the side sheets, are in all cases lower than the actual test critical shearing loads. A part of this difference is undoubtedly due to Vierendeel-truss action of the frame consisting of the upper element, the side posts, and the side plate. It is apparent that a panel designed by the equations discussed in this paper would be suitable for the design loads.

The longitudinal stiffeners were, in all cases, sufficiently large. The longitudinal ribs for test panel No. 2 were, in fact, much larger than necessary as a very small stiffener would have been sufficiently rigid to remain straight while the sheet buckled. It is, however, advisable to make stiffeners more rigid than absolutely necessary, as a panel with ribs with less than the required flexural rigidity would develop tension fields which may be accompanied by permanent bending of the ribs.
An examination of the test results indicates that test panel 
No. 1 with a flat unstiffened 15-gage side sheet had a critical 
-shear load of 9250 lb. Panel No. 2 with 15-gage side sheets and 
two longitudinal stiffeners had a critical shear load of 13,150 lb; 
and panel No. 6 with 15-gage side sheets and six longitudinal 
stiffeners had a critical shear load of 39,400 lb. Thus the applica-
tion of two stiffeners increased the carrying capacity of the 15-
gage sheet only 3900 lb, while the application of six well-designed 
ribs increased the carrying capacity 30,150 lb. Therefore it is 
apparent that a hit-or-miss application of ribs is not necessarily 
of great advantage and that a girder, stiffened with ribs, must be 
carefully designed.

Examination of the summary in Table 1 indicates that panel 
No. 5 is the most efficient panel from the standpoint of critical 
shearing load carried per pound of sheet. However, the load-
carrying capacity of this panel is far greater than that required 
for a 50-ton boxcar, and for economic reasons either panel No. 
6 or panel No. 7 is more desirable. While panel No. 6 has a factor 
of safety of 1.88 which is higher than required, the added sheet 
thickness would increase the time required for corrosion to affect 
seriously the factor of safety.

The critical shearing stresses for all of the test panels were low 
in comparison to the elastic limit of low-carbon steel. The maxi-
mum critical shear stress encountered was the 4000 psi for test 
panels Nos. 4 and 5. Test panels Nos. 4, 5, 6, and 7 are well de-
ed, and it would be uneconomical to stiffen a panel of the 
thickness encountered in boxcar construction to the extent 
required to raise critical shearing stress to that permitted by ma-
terial strength, although the load-carrying capacity of any of 
these panels could have been substantially increased by further 
stiffening. With a given stiffener spacing, the carrying capacity 
of a panel is proportional to the flexural rigidity of the sheet. 
Since the flexural rigidity is practically independent of the com-
position of the steel and is primarily dependent on the plate 
thickness, the critical shearing load of the panels is not increased 
by the utilization of high-tensile material.

Discussion

O. W. Hovey. We are grateful to the authors for presenting 
this paper, and to Dr. Nystrom for authorizing the tests which it 
describes. The ribbed sides of The Milwaukee Road cars have 
been the subject of much observation and comment, and this 
paper shows that their use is based on sound structural theory.

The paper points out the increased resistance to buckling ob-
tained by the addition of the longitudinal ribs, and the consequent 
increase in rigidity of the car sides. It also mentions that the 
ultimate safety of the structure depends on the ability of the 
sheets to act in diagonal tension, which safety factor is retained 
by the effective continuity of the sheets across the lapped joints. 
By introducing longitudinal lapped ribs in the sheets, the posts 
can be spaced farther apart, or thinner sheets can be used, with 
consequent reductions in over-all weight. In fact, with this 
method of construction, the sheets could be considerably thinner 
than those now applied to The Milwaukee Road cars.

In figuring the required section of the stiffening ribs, some 
method must be assumed for evaluating the loads which they 
carry, and thus determining their size. A convenient method is 
to assume that the shear on the car side at any vertical section 
is carried by a number of individual panels equal to the number of 
ribs plus one. As the shear in each of these subpanels has the 
same value vertically and horizontally, a horizontal traction oc-
curs in opposite directions along the two edges of the rib. As-

4 Development Engineer, Alloys Development Company, Pitts-

The authors suggest that with the sheet thicknesses and sizes of 
ribs used in the tests, carbon steel would have resisted the buck-
ling forces as well as high-strength steel. This is true for buckling 
considerations alone, because elastic-stability values are based 
on the modulus of elasticity of the material. It does not hold, 
evertheless, for ultimate values, because the strength of the rib as 
a column is a function of the yield strength of the material, and the 
ultimate shear value of the car side in diagonal tension is a func-
tion of the ultimate strength of the material. Thus the high-
strength steels afford an increased safety factor, or with their 
superior corrosion resistance permit lighter sections without any 
sacrifice in the life expectancy of the car structure.

Author's Closure

The authors wish to thank Mr. Hovey for his discussion and 
for the assistance he has rendered over a period of years during 
the development of the car sides described in this paper.

Mr. Hovey suggests that the stiffening ribs be considered as 
columns eccentrically loaded with the shear load on one side as a 
uniformly distributed compressive load and the shear load on the 
other side as an elastic restraint. The shear load on each side of 
the rib is the total vertical shear load carried by the side times 
the reciprocal of one plus the number of stiffening ribs. Since all of 
the side panels described in this paper are to be designed to carry 
the same vertical shear load it is apparent that the stiffeners for 
panel No. 2 with two stiffeners must, according to the method 
suggested by Mr. Hovey, carry seven thirds of the load carried 
by the stiffener on panel No. 6 with six stiffeners. Thus it ap-
ppears that panel No. 2 requires a larger stiffener than panel No. 6. 
This is not in agreement with the results obtained by the method 
given by S. Timoshenko which is described and applied in the 
paper. According to the method given by Timoshenko the stiffener 
required for Panel No. 2 must have a flexural rigidity of 
only 34,750 lb-sq in. while that for panel No. 6 must be 360,000 
lb-sq in. Since the test results are in reasonably good agree-
ment with the theoretical analysis presented by S. Timoshenko 
the authors cannot accept the method of designing stiffeners 
suggested by Mr. Hovey as reliable. The authors believe that if 
the shear load on one side of the stiffener is considered as a 
uniformly distributed compressive load on the stiffener as a col-
umn, the shear load on the other side of the stiffener which is 
perpendicular in direction and equal in magnitude unload the col-
umn rather than provides an elastic restraint. Therefore, the 
authors recommend that the procedure presented by S. Timo-
shenko, and outlined in the paper, be used for determining the 
proper size of stiffeners. 

The car side section used on The Milwaukee Road is designed 
to carry the car load limit without buckling of the side sheet and 
development of tension ties; this keeps the deflection from becom-
ing excessive. However, as stated by Mr. Hovey, a stiffened side 
section with considerably thinner side sheets than those used on 
The Milwaukee Road could be designed to carry the required 
load by permitting the formation of tension ties and the strength of 
such a side would be dependent upon the yield strength of the 
side sheet material.
Creep and Relaxation in Rubber Products at Elevated Temperatures

By R. D. ANDREWS,1 R. B. MESROBIAN,2 AND A. V. TOBOLSKY3

The studies reported of creep and relaxation at elevated temperatures suggest possible methods of evaluating deteriorative changes occurring in rubbers. Involving intermittent and continuous relaxation and creep measurements, the new methods seem to be readily amenable to molecular-structural interpretations. The practical value of these measurements, apart from their usefulness in fundamental scientific research, is apparent in cases where service deterioration occurs as a result of creep or relaxation. The usefulness and importance of these measurements of creep, relaxation, and modulus are not limited to the measurement of high-temperature oxidative deterioration in rubbers. The simple way in which they are related to molecular changes in the material being studied should make them very valuable in the study of physical changes in polymeric materials in general, such as cold flow of plastics, drift of rubbers, low-temperature stiffening, permanent set, etc.

The creep of rubber products under conditions of constant load and their relaxation of stress under conditions of constant extension are interesting and important problems in themselves in many applications involving the use of natural and synthetic rubbers at elevated temperatures. In addition, recent studies of creep and relaxation at elevated temperatures suggest a new approach to the problem of evaluating the deteriorative changes occurring in rubbers. Although conventional aging tests such as the oxygen-bomb test and the Geer oven test have proved their value as standardization methods, the results of these tests are rather difficult to interpret on the basis of fundamental physicochemical concepts. Intermittent and continuous relaxation and creep measurements, on the other hand, seem to be readily amenable to molecular-structural interpretations.

EXPERIMENTAL METHODS

The apparatus used for measurements of relaxation of stress at constant elongation and also of changes of modulus with time ("intermittent relaxation") has been described in a previous paper, in which illustrations of the apparatus (which was constructed by the Firestone Physics Research Division) are also shown. In principle the apparatus is simply a small beam balance; the rubber samples, which are flat rings (21/8 in. OD, 21/16 in. ID) die out of cured sheet approximately 0.040 in. thick, are looped around a pulley attached to the short end of the balance beam and are extended from outside the temperature box. The moment exerted by the stretched rubber band on the short lever arm is balanced by suspended weights and a rider on the other arm of the beam. During the experiment the length of the stretched band is maintained constant to within 0.2 per cent, because the long lever arm is allowed to swing over only a small angle on either side of the horizontal position. Measurement of the change of modulus with time can also be made with this apparatus, and these studies have been called "intermittent relaxation" studies inasmuch as the measurements are made in exactly the same way as the measurements of continuous relaxation of stress, except that the sample is elongated to the fixed length only momentarily at the time of a measurement, remaining unstretched at all other times.4

Measurements of the creep of samples supported a constant load are made in a very simple way. A frame, shown in Fig. 1, is used, which has four pulleys attached along a crossbar at the top. The samples used are smaller rings (15/8 in. OD, 11/8 in. ID) die from the same cured sheets as the rings used in the relaxation measurements. Pulleys engage the bands at the bottom also and support the weights which are hollow, cylindrical, copper cans whose weight is adjusted by filling with lead shot. The samples are extended between sections of meter stick which are held vertically from the top crossbar. Two pairs of phonograph needles are set diametrically into the lower pulley and bracket the adjacent meter sticks, thus serving to indicate the elongation as well as to keep the samples in position.

1 Frick Chemical Laboratory, Princeton University, Princeton, N. J.
3 Contributed by the Rubber and Plastics Division and presented at the Annual Meeting, New York, N. Y., Nov. 27-Dec. 1, 1944, of The American Society of Mechanical Engineers.
4 The change of modulus with time can also be measured on creep apparatus ("intermittent creep"), by applying the load to the sample only at intervals when a reading is taken, by the method shown in Fig. 1. This method is in general not as convenient as the "intermittent relaxation" method, however.
Theoretical Discussion—Continuous Relaxation of Stress

According to modern structural concepts, soft vulcanized rubbers are three-dimensional networks of long-chain molecules cross-linked by chemical bonds introduced during the vulcanization. The portions of the network between contiguous cross-linking juncture points are known as network chains. The changes that occur in the properties of rubber due to the effects of heat and exposure to air and light are collectively known as aging. It has been repeatedly demonstrated that aging must be considered a chemical reaction occurring in the presence of oxygen. For this reason the rubber bands used in these studies were made sufficiently thin to allow a homogeneous penetration of oxygen.

The results of the studies of continuous and intermittent relaxation and creep have demonstrated that two competing reactions are responsible for the deteriorative changes occurring in rubbers. One of these reactions is a scission of the network chains of the rubber and the other is a cross-linking reaction. Both these reactions must be intimately related inasmuch as they occur with comparable rates over the entire temperature range. This fact suggests that the activation step for both reactions may well be the same.

The measurement of relaxation of stress at constant extension at elevated temperatures provides a means of isolating the scission reaction inasmuch as cross-links tend to form between molecules which are in a relaxed position and so have no effect on the stress. The decay of stress is presumed to be due to the cutting of some chemical bond in the network chains. According to the most satisfactory theory of rubber elasticity yet available, the stress is proportional to the concentration of network chains (or to the concentration of cross-links) in the rubber, and so the rate of decay of stress is proportional to the rate of chain scission.

Fig. 2 shows the results of stress-relaxation studies on five different types of rubber. The experiments were carried out at 100 C, and 50 percent elongation. The ordinate is stress at any given time divided by initial stress, and the abscissa is logarithmic time. For purposes of comparison the same data are replotted in different ways in Figs. 3 and 4.

It is clear from these figures that the different polymers are characterized by quite different relaxation curves. The order of rate of relaxation from the fastest to the slowest is as follows: Neoprene, Hevea, Butyl, Butaprene N, GR-S.

The other facts concerning the relaxation curves are:

1. The rate of relaxation can be slowed a thousandfold by careful exclusion of oxygen.

2. The relaxation curves are relatively independent of elongation up to high elongations.

3. The relaxation curves are not markedly altered by the presence of carbon black in the vulcanizate. In certain cases the presence of carbon black has a slight accelerating effect (see Fig. 5); in Neoprene, however, the presence of carbon black decreases the relaxation rate.
The relaxation curve for Hevea gum follows a simple exponential decay law. The relaxation curves for other polymers can be fitted by a sum of exponential decay terms.

The effect of temperature on the rate of relaxation obeys the Arrhenius law for chemical reactions, namely

\[ k' = A e^{-E/RT} \]  \hspace{1cm} [1]

where \( k' \) is the specific rate, \( E \) is the energy of activation, and \( A \) the so-called frequency factor. Fig. 6 shows the dependence of the relaxation curve on temperature for Hevea gum. The calculated energy of activation for relaxation turns out to be 30.4 kcal in this case.

The effect of the type of vulcanization (e.g., sulphur versus sulphurless cures) is not as important a factor in the stress-relaxation curve as the polymer type.

The presence of certain antioxidants in the vulcanizate definitely retards the rate of relaxation.

Studies of Intermittent Relaxation

Figs. 7 and 8 show the results of so-called intermittent relaxation of stress measurements on Hevea and GR-S gum and tread vulcanizates at 130 C. As previously noted, these measurements are nothing more than periodic measurements of the 50 per cent modulus and are plotted in Figs. 7 and 8, in terms of stress at time \( t \) divided by stress at zero time. It is to be noted that the intermittent relaxation curve for Hevea shows that the modulus initially decreases. In certain experiments where the rubber bands did not rupture, this initial decrease was followed by a subsequent increase. In the case of GR-S, Fig. 8 shows that the modulus increases with time. In these experiments Butyl rubber shows a continuous modulus decrease, whereas Neoprene and Butaprene N, like GR-S, show a continuous modulus increase.

In terms of molecular concepts, the change of modulus with time measures the "net rate" of cross-linking and scission. For this reason the intermittent relaxation curve never decreases as rapidly as the continuous relaxation curve even when scission is predominant as in the case of Butyl and Hevea. The rise of modulus with time in the cases of GR-S, Butaprene N, and Neoprene indicates of course that in these rubbers cross-linking is taking place more rapidly than scission.

Creep Studies

Figs. 9 and 10 show the results of creep studies on Hevea gum at 120 C. Here the creep data are plotted in the form of per cent elongation as a function of linear and logarithmic time. It certainly is reasonable to suppose that the same chemical reactions which are responsible for stress relaxation must be responsible also for creep. A theoretical interpretation of these data\(^4\) has indicated that creep and relaxation should be related as follows

\[ f = \frac{l_0}{l} - \left(\frac{l_0}{l_0}\right)^2 \] \hspace{1cm} [2]

where \( f/f_0 \) represents the fraction of original stress at time \( t \) in a stress-relaxation experiment, \( l \) represents the length at time \( t \) in a creep experiment, \( l_0 \) the initial length and \( l_0 \) the unstretched length.

It is therefore useful to define a creep function, "\( \varepsilon/\varepsilon_0 \)"

\[ \"\varepsilon/\varepsilon_0\" = \frac{l_0}{l} - \left(\frac{l_0}{l_0}\right)^2 \] \hspace{1cm} [3]

Fig. 9 Creep of Hevea Gum at Different Elongations, 120°C

Fig. 10 Creep of Hevea Gum at Different Elongations, 120°C

Fig. 11 Comparison of Creep Function with Continuous Relaxation, Hevea Gum, 120°C

Fig. 12 Comparison of Creep Function with Continuous Relaxation, GR-S Gum, 120°C

Fig. 11 shows the data presented in Figs. 9 and 10, replotted in terms of the creep function against logarithmic time. The relaxation curve at the same temperature is also shown, and it is apparent from these graphs that the creep function is nearly identical with the relaxation function and is largely independent of the initial elongation used in the creep measurement. The same identity of the creep and relaxation functions has been observed for Butyl gum. However, as is shown in Fig. 12, creep is definitely slower than relaxation for GR-S; this is also true for other rubbers which harden rather than soften at elevated temperatures. It has also been observed that the deviation between creep and relaxation is greater in tread stocks than in the corresponding gum stocks. For example, a noticeable deviation between creep and relaxation is observed in Hevea and Butyl tread stocks, though such a deviation does not appear in the gum stocks.

Equation [2] has been derived on the assumption that the observed creep is due only to the scission reaction. This is largely true because at any given moment all cross-links form in a relaxed position. However, as the creep progresses the newly formed cross-linked chains do have a retarding effect. Therefore it is not surprising that in cases where cross-linking is pronounced, the creep function is somewhat slower than the relaxation curve. Despite this effect of cross-linking, however, creep curves of different rubber stocks will show the same general differences as the continuous relaxation curves of the same stocks, and so give a general comparison of the rates of oxidative scission in various rubber stocks.

The practical value of these measurements, apart from their usefulness in fundamental scientific research, is immediately apparent in cases where service deterioration occurs as a result of creep or relaxation. It should be remembered, however, that these measurements are carried out on thin samples in which oxidation is homogeneous. When thick samples are to be used in service, these measurements should be made on samples of the same thickness, for the experimental data to have a direct relation to service deterioration. A particularly interesting practical aspect of these studies is that they show (by the great difference between intermittent and continuous relaxation curves) that the failure of rubber products in service at elevated temperatures will depend largely on whether the rubber is subjected to continuous or only to intermittent deformation. Failure will occur more rapidly under continuous deformation, since failure under those circumstances depends primarily on the scission reaction; under intermittent deformation, failure will result from the net effect of both reactions, and changes will take place more slowly under these circumstances. The behavior of Neoprene illustrates very well the value of making this distinction. Neoprene relaxes and creeps most rapidly of all the various rubber types studied. However, the modulus of Neoprene changes more slowly with time than is the case with most of the other rubber types (as is seen from comparison of their “intermittent relaxation” curves). Therefore, Neoprene will fail very quickly when under continuous load at high temperatures, but should have a longer service life than most other rubbers when used in ap-
applications which involve only momentary intermittent loading.

The usefulness and importance of these measurements of creep, relaxation, and modulus are by no means limited to the measurement of high-temperature oxidative deterioration in rubbers. The simple way in which they are related to molecular changes in the material being studied should make them very valuable in the study of physical changes in polymeric materials in general, such as cold flow of plastics, drift of rubbers, low-temperature stiffening, permanent set, etc., since all these changes in physical properties have their basis in fundamental molecular changes of the type which are reflected in these measurements. Many new applications of these experimental methods will undoubtedly develop with increasing realization of their possibilities.
Investigation of Influence of Ring Size, Bobbin Diameter, and Spindle Speed on Spinning Process, and Their Effect on Over-All Cost of Spinning

By A. N. Sheldon1 and J. J. Blake2

From mathematical formulas and data by Dr. Wilhelm Stiel the authors have developed a series of charts from which may be predetermined, with a fair degree of accuracy, the optimum diameters of ring and bobbin, and speed of spindle, for spinning any size of cotton yarn from any length of staple, carded or combed. The results derived coincide closely with general practice. The use of the charts is explained in some detail by application to specific cases, leading to the prediction of spinning costs under various conditions of spindle speed, ring and bobbin sizes, and the like.

In so far as the authors are informed, there has never been published in the textile literature any chart from which could be predetermined, with approximate accuracy, the optimum diameter of ring and bobbin and speed of spindle, for spinning any size of cotton yarn from any length of staple, carded or combed. In an attempt to fill this hiatus, the graphs which accompany this text have been prepared. The several curves making up Fig. 1 are derived from mathematical formulas and data developed by Dr. Wilhelm Stiel. These have been carefully analyzed and are believed to be correctly deduced and adapted; at any rate, the derivative results appear to coincide rather closely with general practice.

The basic formula is as follows

\[
T = \frac{(2 \times 3.14)^2 \times R_r \times G_l \times N_o^2}{g \times F(\phi) \times 61^2}
\]

in which

- \(T\) = tension in grams
- \(R_r\) = ring radius in meters
- \(G_l\) = weight of traveler in grams
- \(N_o\) = traveler speed or revolutions per minute
- \(g\) = 9.81 (acceleration in meters per second)
- \(F(\phi)\) = factor, depending on spindle speed, coefficient of traveler friction, and ratio of bobbin to ring diameter

Inspection of the chart will readily demonstrate its utility, but probably an exemplary solution of the formula and a brief description of the use of graphs will facilitate their interpretation.

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1 F. P. Sheldon & Son, Providence, R. I. Mem. A.S.M.E.
2 F. P. Sheldon & Son, Providence, R. I.
4 Contributed by the Textile Division and presented at the Annual Meeting, New York, N. Y., Nov. 27-Dec. 1, 1944, of The American Society of Mechanical Engineers.

Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

Application of Equation

For a typical application of the equation, we will consider No. 20's carded yarn spun from 1/16-in. cotton, spindle speed 9000 rpm, with 2V4-in. ring, bobbin diameter 1/3 in., and No. 2/0 traveler weighing 0.0518 g. We will also assume a slip of 3 per cent between spindle speed and traveler speed. Adopting these values in the preceding formula, we have

\[
\begin{align*}
R_r &= 0.0286 \\
G_l &= 0.0518 \\
N_o &= 9000 - 270 = 8730
\end{align*}
\]

These constants supply all of the unknown quantities on the right-hand side of the equation, except \(F(\phi)\), which is determined from the charts, Figs. 2 and 3. From the curve in Fig. 2, we ascertain the coefficient of friction between traveler and ring, designated by the letter \(u\), which, for the example chosen, is 0.23. (For complete data on determination of coefficient of traveler friction, refer to an article by A. Ludicke.) The vertical line at 9000 rpm spindle speed cuts the curve at point 0.23. Then, having found \(u\), we refer to the corresponding curve, 0.23, Fig. 3, on which the abcissa is graduated into divisions representing the ratio of bobbin diameter to ring diameter. In this instance, \(1/u = 0.222\). From this point on the abcissa we raise a perpendicular line until it intersects curve 0.23 and find the corresponding value of \(F(\phi)\) on the left-hand margin to be 4.4.

We now have all of the quantities to be substituted in the fundamental equation for computing the value of \(T\), as follows

\[
T = \frac{(2 \times 3.14)^2 \times 0.0286 \times 0.0518 \times (8730)^2}{9.81 \times 4.4 \times (61)^2} = 27.7 \text{ g}
\]

Referring to Fig. 1, for No. 20's yarn, the scale at the right-hand margin denotes various staples of carded \(K\) and combed \(C\) cottons from which the yarn is spun, while the scale at the left-hand margin shows the safe spinning tension for these staples. The scale at the bottom of the chart gives the ratio of bobbin diameter to ring diameter for the particular condition under consideration.

Suppose, for instance, it is desired to determine the several combinations of ring diameter, bobbin diameter, and spindle speed suitable for commercial 1V4-in. carded cotton. From 1V4 in. \(K\) at the right-hand margin draw a horizontal line to the left-hand margin, indicating a safe tension of 27 to 27.1 g. Any point above this horizontal line indicates an excessive tension, and any point on or below the line, a safe tension to adopt. This line crosses first the curve of 9000 rpm and 1V4-in. bobbin, just above the horizontal line at 27.1 g.
below the 21/4-in. ring, indicating that a 21/4-in. ring, under these conditions, would not be satisfactory. It next crosses the 9000-rpm curve with 1/4-in. bobbin just above the 21/4-in. ring, indicating that 21/4-in. is satisfactory. Next it crosses the 11,000-rpm curve with 1/8-in. bobbin just above the 11/4-in. ring, and then crosses the 9000-rpm curve with 1/8-in. bobbin just above the 21/4-in. ring, indicating that either a 11/4-in. ring at 11,000, or a 21/4-in. ring at 9000 rpm would be satisfactory, with a 1/8-in. bobbin in the first, and a 1/4-in. bobbin the second instance.

Continuing this line from right to left until it crosses the 11,000-rpm curves with 1-in. and 11/4-in. bobbins, we find that a 21/4-in. ring is slightly too large in the first case but satisfactory in the second.

Next, suppose a spinning frame is already equipped with 21/4-in. rings and 1/8-in. bobbins, and it is desired to determine the optimum speed, with 11/4-in. carded staple. First, we drop a vertical line from \( A \) (the 21/4-in. ring on the 9000-rpm curve with 1/8-in. bobbin) to \( B \) (the 21/4-in. ring on the 7000-rpm curve with 1/8-in. bobbin) and note its intersection with the horizontal line through 11/4-in. \( K \), at \( C \). Obviously, the suitable speed, with 21/4-in. ring and 1/8-in. bobbin, lies between \( A \) and \( B \), at point \( C \), which is ascertained by finding the ratio of \( CB \) to \( AB \), that is, as 7 is to 9. Since the difference between 9000 and 7000 is 2000,
Fig. 4 Chart Showing Warp Yarn Strength at 70 Per Cent Relative Humidity, Computed From Formula
\[ \frac{1600}{C (1 + 0.112)} = S \]

(C = counts; S = strength in pounds; \( a \) = difference in sixteenths of staple over or under 1 in.; use + sign when over, − sign when under; \( b \) = difference in number of yarn above or below 28's, use − sign when over 28's and + sign when below.)

Fig. 5 Chart Showing Strength of Combed Warp Yarn, at 70 Per Cent Humidity, Computed From Formula
\[ \frac{1760}{C (1 + 0.112)} = S \]

(C = counts; S = strength in pounds; \( a \) = difference in sixteenths of staple over 1 in.; \( b \) = difference in number of yarn above or below 28's, use − sign when over and + sign when under 28's.)
we add to 7000 the product of 7/9 X 2000, or 1555, and find that 8555 rpm is the optimum speed for the conditions imposed.

**Power Required**

The horsepower required for each ring size and speed, enumerated in the tabulation which follows, has been carefully computed, according to a formula devised by E. A. Untersee,5

\[
Hp \text{ per spindle} = 0.000284 \times R^{1.43} \times \left(\frac{\text{rev}}{1000}\right)^{1.08}
\]

in which \( R \) = ring diameter in inches, and \( \text{rev} \) = speed of spindle per minute.

From subsequent analysis, it is obvious what critical factors are power and fixed charges in the cost of spinning. The conservation of power, in this process, would appear to deserve serious consideration, perhaps by reducing the weight of the spindle through the use of some suitable combination of aluminum and magnesium, perhaps by driving each spindle with an individual motor (similar to the motor drive of rayon spindles), and so eliminate all tapes, tension devices, and cylinders, or by the adoption of both expedients; or by the use of ball-bearing spindles, provided this expedient does not increase fixed charges too much.

With the present mechanism, the power required to drive spinning frames is all dissipated in friction or in heat—more than sufficient to heat the spinning room to a comfortable temperature in winter, even in a northern climate; and which in summer months presents a serious problem in air conditioning. In either case, there is a serious waste of power in the form of heat.

In constructing the chart, Fig. 1, the skein break strength, as determined by our original formulas (charts Figs. 4 and 6), has been divided by 160, to ascertain the corresponding single-strand strength, which in turn has been converted into grams for the scale at the left-hand margin. We realize that this procedure may be subject to criticism, since the average ratio of skein-yarn to single-yarn strength, as determined by numerous tests of a great variety of cottons, is between, say, 100 and 120 to 1, rather than 160 to 1. On the other hand, to insure good running work with a minimum of ends down per hundred spindles per hour, it is necessary to consider the weakest, rather than the average strands, since, obviously, it is the failure of the weakest single strands that controls the result. After reviewing the records of many single-thread and skein break tests, of a wide range of yarns, we have found that dividing the skein break by 160 generally indicates the minimum single-yarn strengths.

In arranging the scale at the left-hand margin of the chart, we have adopted a factor of safety of 10, instead of 14 as suggested by Oertel,4 and as used previously by the authors, as it appears from several mill tests that 10 is a conservative factor to use. That is to say, for computing the single-strand strength, we divided the skein break strength, as determined from our formulas, first by 160, and then by 10.

**Effect on Yarn Tension of Increasing Speed and Ring Size**

To show the effect on the yarn tension of increasing the speed and ring size, the following examples will be cited:

Referring to Fig. 1, for No. 20's yarn, it will be observed that increasing the spindle speed from 7000 to 9000 rpm with a 2'/4-in. ring and 1/8-in. bobbin, the tension is raised from 17.47 to 24.9 g, or 42 per cent; and increasing the speed to 11,000 rpm raises the tension to 34.46 g, or 97 per cent. Again, if the ring size is increased from 1'/4 in. to 2 in. with 1/8-in. bobbin at 9000 rpm, the tension is raised from 18.92 to 23.62 g, or 25 per cent. With a 2'/4-in. ring, the tension is raised to 27.46 g, or 45 per cent, and with a 2'/4-in. ring, the tension is raised to 32.15, or 70 per cent.

Obviously, these variations in tension must affect the quality of the yarn produced. If larger packages are desired in order to reduce the number of piecings, they must be acquired with the acceptance of contingent factors more or less undesirable, and the spinner must choose that expedient which fits his particular necessity best.

It is not pretended that the results deduced herein are of absolute validity, but we believe that the graphs and the method by which they are derived are sufficiently consistent with spinning practice to afford a safe guide to a judicious choice of ring sizes, bobbin diameters, spindle speeds, etc., for most conditions occurring in the spinning process.

Also, while the chart, Fig. 1, embraces bobbins of 1/8 in. diam, we realize that under some circumstances such a small bobbin may be impracticable.

---


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6 "Balloon Shape, Yarn Tension and the Position of the Traveler in the Ring Spinning Machine" (translation of title), by Georg Lindner, Monatschrift fur Textil Industrie, 1910, pp. 213–216.
\[ b = \text{distance from thread guide to maximum displacement of balloon, m} \]
\[ R = \text{radius of ring, m} \]
\[ a = \text{maximum displacement of balloon, m} \]

He also shows that the yarn tension in grams is

\[ T = \frac{N_s^3 \times X^3}{61^2 \times N_s} \left( \frac{90}{180 - \arcsin \left( \frac{R}{a} \right)} \right)^3 \]

which is equivalent to

\[ T = \frac{N_s^3 \times X^3}{61^2 \times N_s} \]

in which \( N_s \) = revolutions of traveler per minute, and \( N_r \) = counts of yarn, English system.

Having determined \( T \), we can predict the weight in grams of traveler required, from the following formula:

\[ G_L = \frac{T \times X \times F(\phi) \times 61^2}{(2 \times 3.1415)^3 \times R_s \times X^3} \]

We have assumed, in each instance, that \( a \) in Fig. 6 equals one half of the gage of spinning frame so that the threads at maximum ballooning theoretically will not collide with each other without separators.

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**TABLE 1** FUNCTIONAL DATA FOR SPINNING NO. 20'S YARN WITH VARIOUS RINGS AND SPEEDS

<table>
<thead>
<tr>
<th>Ring Diam, in.</th>
<th>1/8</th>
<th>1/4</th>
<th>1/4</th>
<th>1/4</th>
<th>1/4</th>
<th>1/4</th>
<th>1/2</th>
<th>2/4</th>
<th>2 1/2</th>
<th>2 1/4</th>
<th>2 3/4</th>
<th>2 1/4</th>
<th>2 1/4</th>
<th>2 1/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spindle Speed, rpm</td>
<td>9000</td>
<td>10000</td>
<td>11000</td>
<td>9000</td>
<td>10000</td>
<td>11000</td>
<td>9000</td>
<td>10000</td>
<td>9000</td>
<td>9000</td>
<td>9000</td>
<td>9000</td>
<td>9000</td>
<td>9000</td>
</tr>
<tr>
<td>( b ) in meters</td>
<td>0.162</td>
<td>0.1462</td>
<td>0.1328</td>
<td>0.1281</td>
<td>0.1462</td>
<td>0.3281</td>
<td>0.3281</td>
<td>0.162</td>
<td>0.1462</td>
<td>0.1328</td>
<td>0.1281</td>
<td>0.1462</td>
<td>0.1328</td>
<td>0.1281</td>
</tr>
<tr>
<td>( R ) in meters</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
<td>10.8</td>
</tr>
<tr>
<td>( a ) in meters</td>
<td>0.273</td>
<td>0.247</td>
<td>0.224</td>
<td>0.207</td>
<td>0.247</td>
<td>0.224</td>
<td>0.207</td>
<td>0.273</td>
<td>0.247</td>
<td>0.224</td>
<td>0.207</td>
<td>0.273</td>
<td>0.247</td>
<td>0.224</td>
</tr>
<tr>
<td>Weight of traveler in grams</td>
<td>0.0735</td>
<td>0.06229</td>
<td>0.05224</td>
<td>0.0472</td>
<td>0.05224</td>
<td>0.0472</td>
<td>0.05224</td>
<td>0.0735</td>
<td>0.06229</td>
<td>0.05224</td>
<td>0.0472</td>
<td>0.05224</td>
<td>0.0472</td>
<td>0.05224</td>
</tr>
<tr>
<td>Approx. weight of traveler in grams</td>
<td>1.2</td>
<td>1.0</td>
<td>0.8</td>
<td>0.8</td>
<td>1.2</td>
<td>0.8</td>
<td>0.8</td>
<td>1.2</td>
<td>1.0</td>
<td>0.8</td>
<td>0.8</td>
<td>1.2</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Approx. number of travelers</td>
<td>2 1/2</td>
<td>1</td>
<td>2 1/2</td>
<td>1</td>
<td>2 1/2</td>
<td>1</td>
<td>2 1/2</td>
<td>3</td>
<td>1 1/2</td>
<td>3</td>
<td>1 1/2</td>
<td>3</td>
<td>1 1/2</td>
<td>3</td>
</tr>
<tr>
<td>Yarn tension in grams</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td>27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2** COMPARATIVE COST OF SPINNING NO. 20'S YARN

<table>
<thead>
<tr>
<th>Ring Diameter - Twist Constant 4:50 - Yarns per inch 30:12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bobbin Diameter:</td>
</tr>
<tr>
<td>Spindle Speed:</td>
</tr>
<tr>
<td>Yds. on Bobbin:</td>
</tr>
<tr>
<td>Lbs. per Spindle:</td>
</tr>
<tr>
<td>No. of Spindles:</td>
</tr>
<tr>
<td>Cost per Wk:</td>
</tr>
<tr>
<td>Spinning</td>
</tr>
<tr>
<td>Winding</td>
</tr>
<tr>
<td>Day Labor</td>
</tr>
<tr>
<td>Power</td>
</tr>
<tr>
<td>Fixed Charges</td>
</tr>
<tr>
<td>Roll Cover</td>
</tr>
<tr>
<td>Add’l Cost</td>
</tr>
<tr>
<td>Add’l Cost</td>
</tr>
<tr>
<td>TOTAL</td>
</tr>
</tbody>
</table>

\( b = \text{distance from thread guide to maximum displacement of}\)

\( R = \text{radius of ring, m} \)

\( a = \text{maximum displacement of balloon, m} \)

Footnote 7, p. 1. 
To demonstrate the validity of the foregoing formulas, we have chosen several examples adopting the maximum traveler speed suggested by some of the machinery builders, as follows:

- 2-in. ring at 11,000 rpm
- 2 1/2-in. ring at 10,000 rpm
- 2 1/3-in. ring at 9000 rpm
- 2 1/4-in. ring at 8000 rpm

and, for comparison, have added

1 1/4-in. ring at 9000, 10,000, and 11,000 rpm
1 1/2-in. ring at 9000, 10,000, and 11,000 rpm

The results are given in Table 1.

Based on Table 1, the comparative cost of spinning No. 20's yarn, from 1 1/2-in. staple carded, would be as given in Table 2, in which the rate of wages per hour is 55 cents for spinners, 61 cents for doffers, 54 cents for winders, and 48 cents for sweepers, cleaners, oilers, and roving men.

Table 2 suggests that for No. 20's yarn made from 1 1/2-in. carded staple, the minimum over-all manufacturing cost is obtained with a spindle speed of 10,000 rpm, 1 1/2-in. ring, 1/4-in. bobbin, and 6 1/2-in. traverse, No. 1 traveler, and 3 1/4-in. gage, without separators. With 1 1/4-in. ring, at the same speed, but with 6 1/2-in. traverse, the cost is only slightly more.

Considered as a group, the use of 1 1/2-in. or 1 1/4-in. rings, at either 9000 or 10,000 rpm, with either 1/4-in. or 1/3-in. bobbins, is apparently more economical than these same rings at 11,000 rpm, or than any of the larger rings. Probably with the use of separators the gage for the 1 1/4-in. and 1 1/2-in. rings could be reduced to, maybe, 3/4-in. or 3 1/4-in.

At 11,000 rpm for 1 1/2-in. and 1 1/4-in. rings, and at the revolutions per minute specified in the tabulation for 2-in. rings and larger, the critical speed of travelers may be exceeded. Consequently, it might be necessary to reduce these spindle speeds appreciably, which in turn would diminish the production per spindle and so increase the number of spindles, and therefore increase the manufacturing costs above the figures given in Table 2.

It will be noted from the foregoing data that the yarn tension and weight of traveler vary as the square of distance b, provided the speed, ring diameter, and bobbin diameter remain constant. The yarn tension varies as the square of the spindle speed, provided the traverse, diameter of ring and bobbin, and distance b remain constant. Also, for 1 1/3-in. and 1 1/2-in. rings, the traverse can be increased 1 in. for each reduction in spindle speed of 1000 rpm.

**Discussion**

F. E. Banfield, Jr.* With nearly half, or about 13,000,000 spindles in place in this country, over 30 years old, the industry is faced with an extensive postwar program of replacement and modernization.

For a number of years, the trend has been toward the use of larger-diameter rings and longer traverses so as to reduce doffing periods, provide longer lengths of yarn with fewer knots or piece ends, and thereby obtain greater economy in subsequent operations. These larger packages can be obtained only at some sacrifice in spinning costs as the authors have indicated. Power requirements are increased approximately by the square of the spindle speed. There is a limit to which the speed of travelers can be run efficiently, and as the speeds are increased the number of ends down are also increased. Therefore, there is a limit to the ring size and speed for a given size and quality of yarn beyond which it is not economical to go. Any savings obtained from larger packages must be weighed carefully against their increased costs.

Reference was made in the paper to the Untersee formula. The object in developing this formula was to provide a means for determining the amount of power that a spinning frame of a given number of spindles, diameter of ring, and spindle speed would require so as to insure the furnishing of the proper size of motor.

Subsequent to working out the formula which the authors have used, Mr. Untersee revised it to include another variable, namely, the yarn size, and at the same time increased the (rpm)² ² ³ power requirement. These larger packages can be obtained only at some sacrifice in spinning costs as the authors have indicated. Power requirements are increased approximately by the square of the number of spindles, diameter of ring, and spindle speed.

This formula was based on data obtained from a large number of tests made under our supervision, not only at our plant but also supplemented by those made in a number of mills under actual operating conditions. This work was carried out by Untersee over a period of years, during which a considerable amount of data were collected; and experience has shown it to be a safe guide to follow.

It is to be regretted that Mr. Untersee's premature death prevented his work from being carried further along lines of including other variables, such as length of traverse, traveler size, etc. However, the formula he has given us is valuable as far as it goes and it is to be hoped that the work will be continued by others interested in the problem. The use which the authors have made of this formula is comparative. For their purpose it would seem to be quite adequate for, as they point out, they do not pretend that the results deduced are of absolute validity but are sufficiently consistent with spinning practice to afford a safe guide to follow in working out this problem.

It should be pointed out that the power consumption varies considerably from empty to full bobbins. The formula takes into consideration the ring diameter only and is based on the bobbins being full. Therefore, the power required for a complete doff from empty to full bobbins will be less than that derived from the formula.

The writer recommends that consideration be given to carrying this investigation further and believes that it is of sufficient importance to the industry for the Textile Division of the Society to establish a committee for this purpose.

C. H. Harrigan.¹ During the last 10 years, the French Ring Spinning Department of the writer's company has made use of improvements such as variable-speed motors, roller-bearing spindles, autolubricated rings, revolving underclearers, balloon controllers synchronized with the ring rail, and our own make of antimarriage (double-spin) preventers, which has made it possible to run many counts and twists of worsted yarn at around 9000 spindle revolutions.

The yarns thus produced are superior to anything previously made here, and the weave shed now demands that all their warp yarns be ring-spun. The mules are now used only for medium counts of yarn that are to be twisted, or for filling.


¹ Forstmann Woolen Co., Garfield, N. J.
We have also overcome the old hurdle of maximum traveler speed by using a metal different from the high-carbon steel commonly used in the trade. In this connection, we are running twisters with 4-in. rings at 7500 spindle revolutions, which is about 1 1/2 miles per m, and travelers last at least 40 hr before changing is necessary.

This answers the conception that high spindle speeds are impractical because of traveler trouble. The writer suggests that improvements in the machines to overcome vibration, such as line shaft and pulleys to replace tin cylinders, lighter metal in spindles, antifriction bearings, so made as to prevent oscillation, and mounting of machines on concrete footings might easily make possible 15,000 revolutions spindle speed, since we have already done 12,500 on 3-in. rings (on a 20-spindle trial frame) and made first-class yarn up to 64's metric.

N. M. Mitchell.11 This subject represents a problem which has confronted mill management from the time ring spinning was invented. In general, spinners and spinning-equipment manufacturers have considered the broad subject one which could best be solved through experimentation.

The activities of industrial and cost engineers in the textile industry require a technical and theoretical approach to the question of what is the proper ring diameter, bobbin size, spindle speed, or traveler for specific counts of yarn, spun from various lengths of staple. In the majority of instances this information is found through analysis of available records covering activities in various mills and experimenting with whatever seems to indicate will best meet the services or requirements for the specific count of yarn under consideration.

Analysis of the present paper indicates the need for giving all factors entering into the formulas extremely careful consideration. The theories presented are generally logical but in practically every instance there is room for criticism from the standpoint of practical spinning operations.

Discussions of the paper with spinners has brought out the opinion that many of the physical and uncontrollable variables normally exist in spinning operations would tend to modify some of the assumptions indicated in the formulas.

There is little to be gained by selecting specific items which have been found controversial in the various tabulations in the paper. The merit in this paper lies in the fact that these progressive and logical steps have been taken, and undoubtedly it was the authors' hope that they would lead the way to further and more detailed investigations which would result in formulas and resultant tables which could be used by the industry as a whole and be found generally acceptable to all concerned.

Brackett Parsons.12 The question of friction between the traveler and the ring is very important in the tension formula of Dr. Stiel. We find that there is considerable variation in the weight of the traveler used, due to the condition of the ring. On a 21-warp yarn spun from 1-in. middling cotton on a 2-in. ring, 11/16-in. bobbin, 8-in. traverse, at 9000 rpm, warp wind, 4-oz package, 3 1/2-in.-gage frame with separators, we have used travelers from No. 3-0 to No. 7, that is, the ring gradually wears, the friction is reduced, and a heavier traveler is required.

The tension formula ignores the effect of different lengths of traverse. This factor in consideration of spinning tension is almost as important as the bobbin-ring ratio. There is, of course, tension variation from top to bottom of stroke, and it naturally follows that the longer the traverse the greater the variation. We also feel that the quality of the roving, the spinning drafts, and spinning-frame construction, that is, the stationary or traversing thread guides, affect the tension limit.

In testing the formula on a limited number of yarns that we are running in sizable quantities, we find considerable deviation from the formula. The formula undoubtedly can be revised to conform with more modern practices.

In considering the cost element, there is a wide variation between our various mills, owing to different methods of manufacture caused by the resultant end use of yarn.

R. W. Voss.13 The approach taken by the authors to the problem of the spinning spindle is most comprehensive and should pave the way for developments of considerable engineering and economic importance. Most of the previous work on the subject has been confined to specialized technical aspects, and comparatively little thought has been given to the interweaving of the results of these detailed investigations into a composite whole which would be of direct utility to the machine builder and the textile manufacturer. While the investigations in the fields of dynamics, friction, vibration, lubrication, and aerodynamics as pertaining to the spindle are of technical importance and interest, it is only by their translation into the actual monetary cost of the daily operation of the spinning that any tangible benefits are obtained.

In gathering together the technical data on which to base the development of their thesis, the authors have been forced to draw from widely scattered sources, covering work done over a considerable span of years and by experimenters of differing modes of approach. This has resulted in the inclusion of both theoretical and empirical results, and of results with considerably varying degrees of approximation. As an illustration, it may be pointed out that Lindner's equation for the shape of the balloon, which the authors quote, is derived in the form of sine curve which is a reasonable approximation to the actual balloon only in cases where the balloon is relatively long and narrow. As the balloon widens out, the sine equation loses its validity just as a parabolic equation loses validity for a catenary, and for precisely the same reason. At the present writing it is not possible to give figures for the degree of this approximation in the case of the large balloon, but it would seem possible that further work since the invention of large balloons is involved in the fouling of adjacent spindles, and hence governs the spindle spacing. A further defect in Lindner's equation comes from the fact that it is derived on the basis of a two-dimensional curve lying in the plane of the spindle. Actually, a balloon is three-dimensional and its backward slope due to air friction further aggravates the departure from the assumed sine curve.

In numerous earlier treatments of the spinning problem, it has been customary to assume that yarn tension is the limiting factor preventing increases in package size and speed. The authors have followed this same general thought but have quite rightly pointed out that, under a certain range of conditions, the limiting factor may be traveler heating and wear rather than yarn tension. Numerous experiments extending over several years' time, made under the writer's observation, tend to indicate that this question of traveler failure is perhaps sufficiently serious to overbalance entirely the consideration of yarn tension under most practical operating conditions. A series of tests on a variable-speed spinning installation designed to give constant tension did not appear to give the full benefit expected, and it was concluded that traveler wear was the disturbing and controlling factor. Other tests, made for an entirely different purpose, showed no particular correlation between yarn tension and spinning end breakage, and in fact showed that yarn breaks due to tension

12 Pepperell Manufacturing Company, Boston, Mass.
13 Director of Research, Chicopee Manufacturing Corporation, Chicopee Falls, Mass. Mem. A.S.M.E.
alone were probably of relatively rare occurrence except in yarn inherently weak. This of course leads to the question as to whether or not the spinning operation should be adjusted to break out weak yarn, but this a matter beyond the scope of the present discussion.

In regard to the actual tension figures used by the authors, it would seem that the values derived from the skin-break test by calculation were open to some question. Tests made in the writer’s laboratory show operating tensions 4 times as high as these, and yet the test was made on a carefully controlled laboratory frame under conditions paralleling operating practice. It would thus seem that, while the use of the skin-break data, figured with an arbitrary factor of safety, might give figures relatively correct, the absolute values might better be determined from actual measurements on the operating spindle. This might lead to extensions of the operating range of speeds or sizes, or both, which would permit distinct economies not shown in the present paper.

An examination of Untersee’s equation for power, and a personal acquaintance with some of Mr. Untersee’s very excellent experimental work, leads the writer to believe that this equation was intended for empirical design purposes rather than for analysis. One defect, in so far as the present application is concerned, is the lack of the inclusion of the length of traverse as a factor. Obviously, the package weight varies with the traverse and this in turn must affect bearing friction. Another defect, from the theoretical viewpoint, is the lack of separate terms for friction, windage, and yarn drag, each of which may vary as a different power of the radius or of the speed. A preliminary study of the fundamental equation indicates that, with conventional proportions of these variables, the equation given by Untersee may hold approximately but, nevertheless, the separate effect of each of these variables is not fully brought out. Since power is such a serious factor it would seem that the equation describing its variation might well be expanded to place it on a par with the calculation of the other elements entering into the total cost of spinning.

The authors have clearly pointed out the high cost of power in the spinning process, and have made a plea for improved mechanical design in the interests of power conservation. This leads to an analysis of the actual power losses occurring in the spindle. There are three causes, namely, bearing friction in the step, windage of the package, and tension of the yarn resisting the rotation of the spindle. The power taken out of the spindle through the yarn tension is ultimately dissipated in traveler friction and in windage of the balloon. It can be shown that each of the three sources of power loss at the spindle is of the same magnitude, and that the loss in the spindle bearing is the only one capable of direct reduction by machine-design improvements. Experimental spindles utilizing rolling contact bearings seem to show a saving in power in this particular location of about one half, but although the bearings have operated satisfactorily over many months’ trial their ultimate life is yet to be determined.

The two remaining items, namely, package windage and yarn tension, would seem to require a completely different method of spinning for their improvement. While windage might conceivably be eliminated by spinning in a vacuum, schemes of this sort seem scarcely practical. In regard to yarn tension, it will be recognized that some amount of tension is necessary to produce a sufficiently firm package, and that as long as this tension is created between the driving spindle and an external drag (namely, the traveler and balloon), the power loss is inescapable. If the tension were supplied through a flyer geared into the driving mechanism of the spindle, this power would be recoverable, but in conventional sizes of spinning spindles it is probable that the friction loss of the added mechanism would render this scheme impractical. The tension is also required in order to keep the balloon within bounds, as the authors have pointed out as a factor which would seem to require inventive genius rather than machine-design ability.

To turn from the engineering aspects of the paper, it may be pointed out that the cost figures of the various operations as quoted by the authors are somewhat out of line with respect to modern mill practice. This is particularly so with regard to the winding operation. However, these matters can be dealt with by relatively straightforward accounting procedures once the engineering fundamentals have been developed, and in any event will vary from one mill to another.

In closing, the writer wishes to compliment the authors for having undertaken this important, but involved, task of correlating the diverse engineering elements of the spinning problem and expressing their result in a form directly usable by the textile industry. The work should prove a guide and an inspiration to those working in the intricacies of mathematical analysis and the delicacies of experimental measurement and those who are prone to forget that their technical results are only part of a broader economic picture.

Authors’ Closure

Before discussing the several commentaries relative to the essential features of our thesis, the authors wish to thank each of the contributors for constructive participation and co-operation in preparing this subject for critical analysis, and, in pointing out the limitations of the formulas and the paucity of accurate empirical data, they recognize the need of further careful exploration.

Mr. Vose quite rightly directs attention to the invalidity of the sine curve where the balloon is excessive in proportion to the traverse, but for conditions usually prevailing, perhaps this critical situation seldom exists. His assertion that the “balloon” curve is three-dimensional rather than two, is equally correct—a fact that Lindner recognizes, although, in his analysis of all the forces involved, he considers the effect of windage more or less canceled out by other effects and takes advantage of this circumstance to simplify what would otherwise be a very inconvenient mathematical expression to use.

The question of traveler performance is another element of the spinning process which, as Mr. Vose observes, requires further intensive investigation, particularly when one considers Mr. Harrigan’s successful experience with travelers made from a special metal and running 40 hr at 132 fps. In so far as the authors are informed, no one except Honegger, has attempted to confirm Ludicke’s experiments to determine the coefficient of traveler friction, and Honegger’s work was, we believe, temporarily discontinued before it was completed.

There is, too, some evidence, confirming Mr. Vose’s test, indicating that spinning end breakages occur, under certain conditions, quite as much from other causes as from excessive tension or weak yarn; for example, the character of the yarn and the fiber from which it is spun.

Tests made by Mr. Vose on No. 30’s yarn, show a safe operating yarn tension of 4 times the value adopted by the authors; and that we can say is that the tensions we have used appear to agree fairly well with results obtained in usual mill practice.

Turning now to the question of power, we are confronted with one of the most significant factors constituting the over-all cost of spinning, and while Untersee has made a notable contribution


\[ 15 \] See Indian Central Cotton Committee "Technological Laboratory Bulletin, Series A, No. 36, Jan., 1937, Matunga, Bombay, India."
toward its determination, his formula does not always coincide with actual practice. Therefore, it is to be hoped, as Mr. Banfield suggests, that his work will be continued by others, in such a manner as to show how the power required to operate spinning frames is divided between all of the moving parts of machines covering a wide range of speeds, ring sizes, traverses, regular bearings and antifriction bearings, etc. It is doubtful if spinners realize how much their over-all costs are affected when they increase speed, ring diameter, and traverse. This investigation of power should embrace the application of variable-speed motors and spinning regulators, to ascertain their economic and practical merits.

Mr. Parsons emphasizes the importance of traveler friction and length of traverse. We have already discussed the former factor; and the effect of the latter is comprehended in the following equations:

\[ h = \frac{180 - \arcsin \frac{R_z}{a}}{90} \]

\[ b = \frac{90}{\arcsin \left( \frac{N_z \times b}{61.5 \times N_s} \right)} \]

He also submits a typical example from actual mill practice; viz., No. 21's carded yarn spun from 1-in. middling cotton, 2-in. ring, 1\(\frac{1}{16}\)-in-diam bobbin, 8-in. traverse, spindle speed 9000 rpm, gage of frame 3\(\frac{1}{8}\) in., and 4-oz. package. Now referring to yarn chart, Fig. 4 of the paper, we find that the skein breaking strength is 81\(\frac{3}{4}\) lb, which is equivalent to a safe spinning tension of 23 g, using a factor of safety of 10. Then from the chart, Fig. 2, we find the coefficient of traveler friction to be 0.24, and from the chart, Fig. 3, \(F\phi = 6\). Similarly, since the traverse is 8 in., \(h\), the distance from pigtail to bottom layer of yarn on the bobbin would probably be 1\(\frac{1}{16}\) in. more, or 9\(\frac{1}{16}\) in., and the sine of the angle obtained by dividing the ring radius by the gage, is 145.15, from which \(b = 5.89\) in., or 0.1496 m. Then, substituting the appropriate values in the applicable formula, we find that the theoretical traveler speed is 8961, compared to the actual spindle speed of 9000 rpm and, likewise, the theoretical traveler weight is 0.0624 g, equivalent to a No. 1 traveler which, it will be noted, is about midway between the maximum and minimum weights actually used, that is, No. 7 and No. 3/0.

In this instance, at least, the computed result and practice agree almost exactly.
Experimental Study of the Flow of Coal in Chutes at Riverside Generating Station

BY E. F. WOLF AND H. L. VON HÖHENLEITEN, BALTIMORE, MD.

Among the power-plant problems which are gradually being solved, the present study of moving coal from the bunkers to stokers or coal-pulverizing mills is important, for with the use of one boiler only for each turbine generator, the prevention of coal stoppages in chutes is vital. During the war years, the use of coal from lot storage, with consequent moisture absorption, has caused a great deal of ratholing and arching in the bunkers, and frequent stoppages have occurred in the chutes. The experimental investigation, reported, involved a study of the flow and stoppage of coal in a transparent scale model of one of the present chutes; measurements of physical properties of lot coal with relation to possible factors affecting stoppage; development in successive steps of a new design of coal chute in scale-model form. Out of this study a suitable chute design was evolved, now being reproduced full scale for installation and test.

POWER-plant operators have been faced for many years with a number of major equipment problems and since some of the more important mechanical difficulties have been overcome, those of a minor nature are now coming to the front. At the same time the constant improvements in the art of boiler design have magnified some of these minor troubles. Getting coal from the bunker to stoker hoppers or to coal-pulverizing mills has been one of these problems. Heretofore the operator has usually been successful in keeping fairly dry coal flowing through his chutes, or in relieving stoppages due to wet coal by employing air lances and some judicious pounding at points where such stoppages occurred. With the use of one boiler for each turbine generator, the prevention of coal stoppage now assumes major importance.

When the Consolidated Gas Electric Light and Power Company began its expansion program in 1939, it recognized the importance of a trouble-free coal supply and surveyed the field to determine the latest advances in the art of coal-handling and chute design. The chutes installed at the Westport Station extension and at the new Riverside Generating Station represented the best engineering knowledge available up to that time. Chutes, in general, were kept at as steep an angle as possible, flared chutes were employed, and the angles of the sides of hoppers were maintained at 60 deg or steeper. This has resulted in a chute design, which, in general, has performed well with moderately dry coal.

During the winter and early spring of 1943-1944, government regulations of fuel made it necessary to use large amounts of fuel reserves. This led to extensive use of coal from lot storage, which by reason of exposure to weather had absorbed large percentages of moisture. With this wet coal a great amount of ratholing and arching took place in the bunkers and frequent stoppages occurred in the chutes from the bunker to the pulverizing mills, especially at the Riverside generating station. It was decided that a thorough study of this problem would be warranted and an experimental program was mapped out and carried through in the following steps:

1. Study of the flow and stoppage of coal in a transparent-plastic scale model of one of the present chutes.
2. Measurements of physical properties of lot coal to study the cause and factors affecting stoppage.
3. The development in successive steps of a new design of coal chute in scale-model form.

The purpose of these experiments was to find a means by which the flow of coal through the chutes could be improved with a minimum of changes. In the redesign of the chutes themselves, extensive structural or mechanical changes could not be contemplated since they would necessitate prolonged outages of plant and their cost would be prohibitive. This decision put certain limitations on the development of a new design and made it impossible therefore to obtain the ultimate in performance.

The experimental work led to designs which in scale form handle, without stoppage, coal of 11/4 times as high a moisture content as the model of the existing chute. The final model permits unimpeded flow of coal at a moisture content which ratholes, arches, and sticks in the model of the existing bunker. A full-sized coal chute conforming to the final model design will be placed in operation early this spring and will serve to check results obtained in the laboratory.

STATEMENT OF PROBLEM

It has been observed over a number of years that coal will stick in bunkers and chutes whenever the moisture content exceeds a certain limit for given types of equipment and coal. Stoppages, when using lot-storage coal, have been more frequent at our Riverside plant than at our other plants, principally on account of the method of storing coal at the former location.

![Fig. 1 Sieve Analyses of Coal](Representative samples taken from snow deliveries and from Riverside storage lot.)

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1 Consolidated Gas Electric Light and Power Co. of Baltimore.
2 Contributed by the Fuels and Power Divisions of The American Society of Mechanical Engineers and presented before the Boston Section on April 12, 1945, and before the Metropolitan Section on May 28, 1945.
3 Notes: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.
The stocking-out and reclaiming of coal by means of a bulldozer and "carry-all," and the greater degree of compacting that the coal undergoes when stored by this means, is apparently responsible for a higher percentage of fines and, consequently, a greater ability of the coal to retain moisture.

The fuel used at these stations is semibituminous coal produced in District No. 1 from the central Pennsylvania and northern West Virginia regions.

Practical operating experience had indicated that the coal delivered to our plants by barges normally could be relied upon to flow through the chute system with little difficulty in contrast to troubles experienced with lot-stored coal of similar moisture content. It was believed that the explanation for this was to be found in the physical make-up of the coal. Typical sieve analyses plotted in Fig. 1 indicate that the consistency of lot coal is such that approximately 30 per cent by weight is finer than number 20 mesh, and that it contains approximately one third more material of this fineness than scow coal.

Each of the two turbine generators at Riverside Station is served by one 550,000-lb per hr boiler which takes its coal supply from a 990-ton-capacity catenary bunker. The bottom of this bunker is located at elevation 72 and is equipped with six clamshell gates. Three chutes, each fed by two gates, carry the coal to the three pulverizing mills located in the boilerhouse basement at elevation 10, Fig. 2. The center chute, which had proved to be the most troublesome in operation, was selected as the subject for this investigation.

Study of Flow of Coal in Scale Model of Existing Chute

Since it was impossible to determine either the exact location or the nature of each stoppage in the coal chutes at the plant, it was decided to make an exact transparent scale model of one of the existing chutes. This and most of the later models were fabricated from 30-mil sheet pyralin by molding it under boiling water over polished wood templates which conformed to exact inside dimensions. This material is satisfactory for this purpose since it has a coefficient of friction not greatly different from smooth steel plate. The linear scale selected was one tenth full size, which is equivalent to a volume reduction of 1000 to 1. Experiments were conducted with this model using specially prepared coal samples having particle sizes reduced approximately to the linear-scale reduction of the model. Most of the experiments were carried out, however, with coal samples taken directly from lot storage and slightly modified by removal of lumps larger than \( \frac{1}{2} \) in. size. A revolving-table feeder was installed at the outlet of the model to simulate the action of the table feeder at the Riverside plant. Phenomena observed in the actual chutes have been reproduced to a surprising degree in the model with either gradation of coal. Sticking and "hanging up" have occurred in the model at the same location and in the same manner at which sticking could be observed in the prototype.

The authors' previous experience (1, 2) had demonstrated that a scale model is a useful tool in studying certain power-plant design problems, and careful consideration was given to the reliability of model tests for this specific application. It was recognized that a considerable scale effect exists but, since the components were too numerous and too much subject to variation, no attempt was made to arrive at a mathematical expression of

\footnote{Numbers in parentheses refer to the Bibliography at the end of the paper.}
the scale factor. While it may be possible to determine such a factor for one part of a system, it seems a hopeless task to establish it for all component parts or to arrive at an over-all scale factor. It was determined, however, that all the phenomena which had been noted in the actual coal chute at an observed moisture content of between 5.5 per cent and 6 per cent occurred in the model between 3.5 per cent and 3.75 per cent moisture content. It is recognized therefore that a scale factor exists but that the scale effect will tend to give model results on the conservative side.

In the operation of the model of the existing chute, the flow of dry coal was found to be smooth and uniform, which is in accord with service experience. At about 2.5 per cent moisture, the flow became very sluggish along the corner of least slope in the hopper below the operating level (elevation 31 ft 6 in., Fig. 2). Most of the flow occurred in a spiral motion along the opposite vertical corner. The movement of coal from the chute above occurred almost entirely on the side where the hopper below was offset. With further increase in moisture the coal began to rat-hole intermittently by draining entirely from the vertical corner and leaving the remainder of the hopper packed with coal. The observation of these phenomena which have been noted fre-
sequently in the field, was aided in the model by the use of thin horizontal layers of inert white powder placed at intervals in the coal stream. Two other early points of trouble were the bottom of this hopper and the hopperlike shape simulating the coal gate.

In the range from 3.0 to 3.5 per cent moisture, stoppages occurred in the latter locations and voids began to form in the rectangular tapering chute above the operating floor. With further increase in moisture, flow of coal could be maintained through the lower hopper by means of scale-sized pokers and air lances, but arching and ratholing then took place at the upper hopper, Figs. 3 to 7, inclusive.

In this chute system, there is a reduction in cross section from a rectangular shape at the top of 41.7 sq ft to a circular shape at the bottom of 1.7 sq ft. This large transition causes the coal to compact, producing the observed stoppages at the bottoms of the hoppers, in corners and at points of change in direction of flow.
PHYSICAL PROPERTIES OF COAL

To determine the cause of the stoppages which were observed in the model and in order to effect improvements in design, it was found necessary to make quantitative measurements of those physical properties which seem to govern the flow of coal. The ratholing, arching, and sticking were found to be closely associated with three mutually interrelated characteristics, namely, moisture content, degree of compacting, and uniformity coefficient. The gradation of the coal affects the amount of moisture which can be retained and also the possible degree of compacting.

Numerous measurements were made with simple laboratory equipment devised for this purpose. The size of specimen was in the range of 20 to 50 lb in most cases. In these discussions the term "loose coal" is being used to indicate that the coal was placed loosely, one scoopful at a time, and every attempt was made to avoid compacting. The term "fully compacted coal" implies that the container was filled with coal and vibrated by mechanical means until no further change in volume could be observed.

Static Angle of Repose. One of the properties which governs the gravity flow of coal is the static angle of repose which is influenced by both moisture content and degree of compacting.

For the determination of this static angle of repose a 12-in. X 12-in. X 12-in. metal box with one side hinged at the bottom was used. The box was first filled with loose coal, then the hinged side was dropped, and after all loose coal had fallen away the angle of the cleavage plane was observed. The box was then filled with coal and vibrated until full compactness was reached. Again the side was dropped and the cleavage angle noted. This test was repeated for moisture contents ranging from 1 per cent to 18 per cent and the results are shown in Fig. 8. With fully compacted coal, the angle of repose increases rapidly and reaches 90 deg at 3.6 per cent moisture. Consequently, at this and higher moisture contents, ratholing can occur in hoppers and bunkers. With an increase in moisture content above 3.6 per cent, it was possible to undercut the coal. At higher moisture ranges (above 13 per cent), the wet mass of fully compacted coal had a tendency to bulge out. It may be stated therefore that the angle of repose for these conditions exceeds 90 deg, or, wording it differ-
Density of Loose and Fully Compacted Coal With Various Moisture Contents. The 1-cu-ft box used in previous tests was filled repeatedly with coal of various moisture content, and the weight per cubic foot checked. These weights have been plotted for loose and fully compacted coal in Fig. 9. Starting from 1 per cent moisture content, the density of loose coal decreases until it reaches its lowest value at approximately 7 per cent. Above 7 per cent, it rises until at 15.75 per cent it again reaches the same density as at 1 per cent moisture and continues to rise. Some other granular materials, for instance, sand (3), show this same phenomenon of a point of minimum density at an intermediate moisture content.

Effect of Moisture Content on Degree of Compacting of Coal. To gain a better understanding of the interrelation of moisture content and degree of compacting, additional tests were made, and the information is plotted in Fig. 10. It will be noted that a definite relation exists between the lines on this graph and the information given in Figs. 8 and 9. With very low moisture contents, coal can undergo full compacting without reaching an angle of repose of 90 deg and, consequently, with such coal, ratholing is highly improbable. At 3.6 per cent moisture content and above, there is a definite degree of compacting that can take place before an angle of repose of 90 deg is reached. With increasing moisture content, this margin diminishes.

Dynamic Angle of Repose. The tests described represent the angle of repose under static conditions which will differ from the angle of repose under dynamic conditions. Another series of tests was carried out in which the coal was permitted to fall through a 2-in-sq opening in the bottom of the 1-cu-ft box. The angle of the cone formed by the loosely falling coal is plotted in Fig. 11.

Angle of Slide of Coal With One Per Cent to 16 Per Cent Moisture Content on Various Metal and Nonmetallic Surfaces. The suita-
bility of various materials for coal chutes was studied. The angle at which coal will slide off these various materials is one indication.

A large plate was arranged in such a fashion that it could be raised slowly from the horizontal position. Various materials were attached to this plate and equal amounts of loose coal carefully placed along the top part of each material, Fig. 12. The angles of slide are plotted in Fig. 13. The variations in slide angle of the materials tested were within 3 deg with coal up to a moisture content of 8 per cent. Above this point the graph shows breaks in the curves for glass, aluminum, and stainless steel. This change in characteristics is apparently due to the formation of a film of moisture on the surface of the material which permits the entire specimen of coal to slide as one mass.

These slide tests represent unrestricted flow with one surface of contact. The influence of three contact surfaces was determined by using channel-shaped metal chutes, Fig. 14. This figure shows that the slide-angle curve for steel also has a maximum value.

A series of experiments was also performed with cylindrical pyralin, glass, and steel tubes to determine the variations between the foregoing conditions and one in which no free surface of coal exists.

Test points have been incorporated on the graph shown in Fig. 15, and an average curve has been drawn to show the trend. It will be noted that the angle of flow increases until it reaches a maximum at 10 per cent moisture and decreases thereafter.

At least three types of flow have been observed in these experiments, which account for the nonuniform behavior in the intermediate moisture ranges, as shown in Fig. 15, and are as follows:

Granular flow, in which there is distinct movement of one particle with respect to the others, occurs with fairly dry coal.

Plug flow, in which the whole column of coal moves as one mass with no apparent relative movement of individual particles, occurs at high moisture contents.

With moisture contents corresponding to the trough of the “density-moisture curve,” Fig. 9, there exists a transition type of flow which resembles viscous flow.

The transition from granular flow to plug flow cannot be clearly defined and is influenced by the physical composition of coal. In this region of change in type of flow, it was difficult to obtain reproducibility in many tests.

Flow of Coal in Pipes With Uniform and Enlarging Diameters.

The wisdom of using pipes with constant increase in cross section in direction of flow (flaring pipes) has been long recognized, and measurements were made to establish the relative merit of a flaring pipe as compared with a pipe of uniform cross section. A piece of smooth steel tubing 9 ft in length and of a uniform 3-in. ID was set up at an angle of 60 deg and paralleled by a pipe of the same length but gradually enlarging from 3 to 4 in. ID. These pipes were carefully filled from the top and the coal was taken away from the bottom by means of a slowly rotating disk. The stoppage occurred in the 3-in. cylindrical pipe at 7 per cent moisture, while the tapering pipe was capable of carrying coal of any moisture content up to the limits of the test (16 per cent).

Effect of Partial Obstructions of Outlets to Coal Chutes. Observations were made on the effect of small obstructions on the
angle of slide in cylindrical steel tubes. It was noted that, above 4 per cent moisture content, a very small restriction at the end of a pipe, either in the shape of a small segment, or in the shape of a minute annular ledge, was sufficient to raise the angle of slide to 90 deg, or even to prevent flow in a vertical position. This observation is particularly important since it indicates that a very small projection at a flanged joint can be the cause of stoppage.

**Side-Wall Support of Column of Coal by Smooth Cylindrical Pipe.**
During the model tests, the vertical load that the coal exerted at the bottom of the chute system was measured. It was found to be equal approximately to the weight of a column of coal 1 diam in height. Basic tests were made to determine the influence of a given head of coal in producing flow through a chute. For this purpose, a smooth 3-in-diam steel cylinder, 36 in. long, was arranged vertically in such manner that the bottom of the pipe was suspended over the platform of a scale without actually bearing on it. This pipe was filled in successive steps to varying heights and tests repeated with samples of varying moisture content. At each step the weight of the coal borne by the scale was read to 0.01 lb. The difference between this weight and the total weight of the coal in the pipe represented the support afforded by friction of the wall of the cylinder. It is significant that all curves, plotted in Fig. 16, show a fairly uniform rise up to 1 diam and then flatten out considerably, showing a very small gain in pressure on the bottom from additional height of column. With increasing moisture contents, each successive curve shows lower values and, after having reached minimum values at 8 per cent, begins to rise. This is in close relation with Fig. 9, which shows the change in density of coal with varying moisture contents. With fairly dry coal, approximately 80 per cent of the weight of the 36-in-high column is carried by static side-wall friction.

**Constricting Chutes or Hoppers.**
In the model of the existing chute as well as in the field, it has been observed that the hoppers were focal points of trouble. For the development of a new design, it became necessary to assemble basic data on the flow of coal through hoppers with various degrees of constriction, shape, and size.

The curve representing the moisture content at stoppage in cones of varying tapers increases rapidly with the increase in steepness of cones, Fig. 17. With the sharp-angle cones it was possible to re-establish flow by further increase in moisture content. The pronounced effect of compacting on the stoppage of coal in hoppers may be seen by a comparison of the two curves in this figure.
A number of eccentric cones with one vertical side were also investigated but it was found that, for equal change in cross section, the concentric cone gave slightly better results.

In order to obtain further indications on the trend of the scale effect, three cones of equal taper (1 in. per ft) but with successively increasing dimensions were also investigated. For this series it was found that a cone having 2 in. ID at the small end, a height of 12 in. and 4 in. ID at the large end would produce stoppage with loose, 7 per cent moisture coal. The next larger cone with 4-in. base and 8-in. top diam, 24 in. high, stuck at 9 per cent moisture, and the largest cone, varying from 8 in. to 12 in. ID, 36 in. high, failed to function at 11 per cent moisture.

**Effect of Addition of Oil and Wetting Agents on Flowing Properties of Coal.** The use of admixtures to lubricate the sides of a chute was considered and various experiments were made to establish the effect of oils and wetting agents. No appreciable improvement of practical value was obtained with these admixtures.
REVISED MODEL DESIGN

With the basic data available as explained, it was decided that improvement to the existing chute system could be accomplished without change in the arrangement and controls of feeders or without alterations of any main structural members or mechanical equipment.

From the shape of the curve showing variations in moisture content of coal during the operating year, Fig. 18, it may be seen that a moderate improvement in chute performance actually represents a considerable decrease in the number of days in which trouble may be expected. It was with this thought in mind that the redesign of the experimental coal chute was begun.

Successive steps of new design incorporating certain features, to be mentioned, were tried in model form. Essentially these features are intended to approach streamlined flow of the coal similar to the flow of fluids in an efficiently designed system and are:

**Fig. 21** "FULLY STREAMLINED" MODEL
(Lateral transition above operating level accomplished by reverse bend in flaring chute.)

**Fig. 22** MODIFIED "STREAMLINED" MODEL
(Lateral transition above operating level accomplished by single bend in flaring chute.)
1. Avoidance of sudden constrictions and sharp changes in direction.
2. Minimum angles of convergence of lines of flow, preferably approaching zero.
4. Maximum possible angle of inclination with horizontal throughout the system.
5. Use of round shapes in preference to square or rectangular shapes.

Since existing space limitations in the plant had to be taken into consideration and the design modified to suit these conditions, it was not possible to apply fully the ideals outlined.

**Description of Design Changes as Tested.** The first problem encountered was the hopper below the operating floor (elevation 31 ft 6 in., Fig. 2). In the present chute system, this hopper forces the coal into a two-directional lateral transition. This double transition causes the greater part of the hopper space to be inactive. The first changes that were tried were the elimination...
Fig. 25 Example of Symmetrical Conical Hopper Sections
(Long cones of slight taper used to accomplish convergence of flow and transition to single pipe. Difficult to relieve stoppages occurring in long cones.)

Fig. 26 Example of Use of Offset Cones
(A study to determine behavior of offset cones as a means of accomplishing lateral transition.)
of one transition in the hopper by shifting it to the chutes above, and the use of a conical hopper of small taper, Fig. 19. Further alterations led to a fully streamlined hopper with a lateral transition in one plane only, Fig. 20.

From the underside of the coal bunker to the operating level, the design possibilities were limited by the following facts: The existing coal gates for this system of chutes are spaced 8 ft apart and cannot be changed. The location of the top of the hopper below the operating floor has already been set by virtue of existing structural members and various piping below this floor. In addition to this, there are some main structural members approximately halfway between the operating floor and the bunker.

To accomplish the necessary lateral transition, a number of flared circular sections were tried, first with a reverse bend, Fig. 21, and then with a single bend and straight section, Fig. 22.

Above this section of pipe, the problem still existed of bringing two streams of coal together, and a series of successive shapes led to the development of the completely streamlined model shown in Fig. 21.

Altogether 40 setups were tested covering the full range from a large long hopper section spanning over both bunker gates at the top, Fig. 23, to the extreme of two pipes leading from the coal gates in the most direct manner possible down to the hopper at the operating floor, Fig. 24. These various setups are in part depicted in Figs. 25 to 27, inclusive.

The fully streamlined model which had been assembled and tested to explore the theoretical solution to the problem was not regarded as practical from an engineering standpoint. Since nearly every component part consists of double-curvature shapes, the difficulties encountered in building the model might easily be multiplied in the actual fabrication of a prototype. The model shapes became so sensitive that the slight distortion which the plastic pyralin model underwent during hot humid weather immediately reacted on the ability of the system to carry wet coal. Considerable effort was devoted to the problem of simplifying these shapes to a point where manufacture would be feasible and operating effectiveness would be maintained. The final results of these tests are shown in Figs. 28 and 29, which represent essentially the chute that has been ordered for a full-scale test in the plant at Riverside.

**Discussion of Special Shapes and Problems.** Several individual problems of interest to the designer were encountered in these tests. In the model of the existing chute, the coal had a tendency to hang up at the transition from the rectangular chute to the hopper below the operating level. This difficulty was especially noticeable where the flow from the rectangular chute was directed against a slanting side of the hopper below. Where this hopper stepped back, permitting the coal to fall away from the end of the chute, less trouble was encountered. This observation led to the development of the "breakaway," which consists of a free space surrounding the lower end of a chute or hopper, thereby permitting the coal flowing from the chute or hopper to break away freely in all directions, Fig. 30. This feature permits a sudden change in direction and has the added advantage of tending to counteract compacting. This device has been employed either wholly or partially in three locations in the new chute system.

The data gained in the cone experiments indicate that the behavior of a separate conical hopper will differ from one incorporated in a chute system. Tests carried out in the development of the new model further indicate that the performance of such a hopper depends on its relative position within a system, since this will govern the amount of compacting the coal undergoes. In general it may be stated that cones of a slight taper, while permitting free flow as an individual unit, show a tendency to compact the coal by their powerful wedge action when they become part of the system, and whenever the ratio of their lengths to smallest diameter exceeds approximately 2. On the other hand, short cones of moderate slope work fairly well as long as the ratio of their lengths to smallest diameter does not exceed 1.5 to 2. Concentric cones have an advantage where it is desired to achieve a reduction in cross section while eccentric cones can be employed effectively where both lateral transition and reduction in area are required. Consequently, both have been employed in the final design.

**Conclusions**

The results of observations given in this paper apply to the particular coal tested and, for similar coal the graphs should be interpreted as indicating probable trends, rather than definite values.

The tests have demonstrated that phenomena occurring in a chute system can be reproduced in a scale model. Application of principles developed in these experiments can lead to improved chute design.

Compaction of coal is one of the most important factors influencing the operation of a solidly filled chute system. An angle of repose of 90 deg may be reached at relatively low moisture contents if the density of coal is increased through impact or wedge action. Since coal must undergo a certain amount of compacting in any composite system, there exists a limit in moisture content at which the chutes can be depended upon to operate without stoppage. The possibility of coal stoppage must be recognized, and therefore the ease of re-establishing flow should be considered in the development of a design.
Fig. 28 Model of Final Design
(Upper hopper and converging section fabricated from pyralin-lined tinplate.)

Fig. 29 Side View of Final Design HopperBelow Operating Level
(Offset at upper portion was necessitated by obstructions.)
It was concluded that the following points should be incorporated in the chute design. The shortest possible path from the coal bunker to the mills should be maintained. Tapered (or uniformly enlarging) round chutes should be used wherever possible. Where two streams of coal must be brought together, this should be accomplished with a minimum angle of convergence. If a sudden change in direction cannot be avoided, the introduction of a "breakaway" to help the coal to realign itself should be utilized. All angles of chutes should be made as steep as possible and reductions in cross section kept to a minimum.

The application of all points mentioned in the foregoing has resulted in a model design which showed sufficient improvement over the model of the existing chute to justify the construction of a full-sized chute for development of actual field experience, Fig. 31.

This investigation has concerned itself entirely with the possibilities of improving flow of coal in an existing chute system and does not take into account the problem of maintaining uninterrupted flow of coal from bunker to chutes, but the latter is now under investigation.

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(Owing to travel emergency conditions existing when this paper was presented, discussion will be accepted until November 10, 1945)